THESIS

ON

"THE EFFECT OF ADVERTISING ON SALES"

SUBMITTED BY

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PREFACE

The Thesis on "The Effect of Advertising on Sales" consists of seven parts. Part I introduces the subject. Part II reviews and critically evaluates the work done in this area. Part III deals with materials and methods. Part IV deals with observations. Part V deals with discussions. Part VI deals with conclusions. Part VII summarises the thesis. Then follows a bibliography of the works consulted, and an additional bibliography.

ACKNOWLEDGMENTS

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PART I

I N T R O D U C T I O N

I N T R O D U C T I O N

"The sales effect of advertising is like the unicorn or mermaid - something we all have heard of but few of us ever expect to see. If it exists, it is in a realm as remote as Camelot".[1]

The literature of the advertising - sales relationship is epitomised in this statement. All the "how-to-measure-advertising-effectiveness" pronouncements notwithstanding, the plain fact of the matter is that a direct, causal, relationship between advertising and sales has yet to be established and measured in a satisfactory manner.

To the line executive whose managerial responsibilities may not allow an intimate association with the workings of advertising, this conclusion may seem rash, why can't the sales effect of advertising be measured? What about all the research on this problem? Has management been the victim of a hoax? Straightforward answers are called for.

A PROBLEM OF CAUSE-AND-EFFECT

"Relating sales results to advertising is basically a problem in establishing a causal relationship."[2] When dealing with social

phenomena, the effects of causes cannot be seen in the same sense as the physical world. For example, a chemist can see the combustion when he adds sodium to water, but the market researcher cannot literally see a consumer attitude being changed by a Radio Commercial. Since these effects cannot always be observed, they may have to be inferred. What are the bases for inferring a causal relationship?

To assert that sales are caused by advertising implies that, if advertising is varied while all other factors that might effect sales are held constant, sales will vary in a predictable way. To establish this causal relationship requires that both its direction and magnitude be specified. "Direction" means that advertising must be the cause, and sales, the effect. Even though it seems intentively obvious that advertising causes sales, consider the rather common business practice of determining the advertising appropriation according to some percentage of - sales method. "Magnitude" refers to the extent to which advertising causes sales - that is, one Rupee spent on advertising brings about how many Rupees (or units) in sales? Obviously, the direction of the relationship must be known before its magnitude can be measured. In practice, both these dimensions have given researchers headaches.

Three types of evidence serve as the basis for establishing a causal relationship: (1) concomitant variation, (2) time order of occurrence, and (3) elimination of other possible causal factors. In the advertising sales relationship, "concomitant variation"

means that, if advertising causes sales, a higher level of sales would be observed in the presence of advertising than in its absence. Similarly, a larger change (increase or decrease) in sales would be expected to result from a major change in the level of advertising than from a minor one. "Time order of occurrence" means that, if advertising brings about sales, it must precede sales. Finally, "elimination of other possible causal factors" means that, in order to assess the effect of advertising on sales, all other factors that might simultaneously affect sales must be accounted for.

To specify both direction and magnitudes that is, to establish a cause - and - effect relationship, the analyst brings to bear three types of evidence. First, he determines whether the supposed cause varies with the effect. As a matter of procedure, the investigation may begin either with the presumed cause (and work "forward" to the resultant effect) or with the sought for effect (and trace it "back" to its cause). Second, the notion of concomitant variation is used to determine whether advertising is at all related to sales results. If, by this criterion, it is related, time order of occurrence is used to specify the direction of the relationship, that is, it must be demonstrated that advertising, "comes first". Finally, the magnitude of advertising's affect on sales would be determined by the elimination of other possible causal factors.

The conceptual rules for determining that sales are caused by advertising are quite specific, however, they by no means assure that, in real world application, the cause-and-affect relationship

can be established in a satisfactory manner. Indeed, it has not.

ADVERTISINGS PECULIAR PROBLEM

As every reflective manager knows, advertising functions in an environment in which many other factors may affect sales. Thus to assert that a consumer purchased brand X as a result of a Radio Commercial is to conclude that he or she was oblivious to every other sales - affecting factor. Such a conclusion would be based on impossible assumptions. No one - including the consumer - can be certain that the design of the product, its brand image, its price, the circumstances surrounding its purchase, and other previous and simultaneous advertising did not influence the selection of the brand. Moreover, the extent of these influences is uncertain. To establish a causal relationship in the sense described above under these circumstances is no small task.

Consider some specific elements of the problem of determining the volume of sales resulting from a unit of advertising. Although, typically, demonstrating that advertising and sales vary concomitantly is not difficult, caution must be exercised in the interpretation of this relationship. Correlation analysis is useful for the purpose of establishing the degree of direct, or inverse, association among variables; it does not establish cause, however. Thus, a high degree of correlation between sales and advertising actually proves nothing. By definition, in fact, simple correlation coefficients are symmetric, that is, the correlation between sales

and advertising is the same as the correlation between advertising and sales. Therefore, in so far as facts are concerned, the conclusion that sales cause advertising is no more incorrect than the conclusion that advertising causes sales. Further, when interpreting a measure of correlation, there is little certainity that spurious relationships are not distorting the true state of affairs. Most students have chuckled at the high degree of positive correlation between sales and advertising, but it should be borne in mind that this positive correlation may not be due to advertising, but may be due to very favourable competitive advantage that overcomes the possibly detrimental effect of advertising. This statement merely reflects the fact that a simple correlation coefficient is a summary measure of the degree of linear association between two variables. An analytic procedure for estimating this degree of association while simultaneously holding constant the effects of other variables is to compute the partial correlation coefficient. Even this procedure has limitations, however, its usefulness depends on the extent to which the relevant variables are discovered and held constant.

The difficulty posed by crude approximations of advertisings effect on sales is that management may be bulled into a false sense, of security. To accept the proposition that advertisings' effect has been measured because the firm's sales increased when advertising was increased, is to be dangerously misinformed. Indeed, it may be far more prudent to formulate advertising strategies with full

knowledge of the risks imposed by the unknown than to venture forth into the competitive struggle confident - but wrong - about the effects of advertising.

STATUS OF RESEARCH EFFORTS

The purpose of industry's annual investment of lakhs of Rupees in advertising is, of course, to favourably affect consumer purchase behavior - either in the aggregate or for the competitive benefit of various firms. In either case, it is of no small consequence to a business firm to be able to ascribe to its advertising a reliably quantitative sales result. Despite the large amount of research directed to this end over the years, it has not been achieved. Without denying the many ingenious research efforts or disputing their conclusions, the unpleasant fact remains that there is no known universal procedure for quantitatively measuring the singular sales effect of a unit of advertising. (Conceivably such a procedure could exist under the cloak of competitive secrecy, but it is hardly likely).

The efficacy of existing advertising - sales models depends on the extent to which their assumptions are satisfied. These models have been developed - their relationships formulated - under certain assumptions. When the models are applied, therefore, these same assumptions must hold true lest the advertising - sales approximation lose all significance. For example, some models

apply only to certain kinds of products - for instance, those purchased frequently. Others assume that all advertising is of equal quality, whether in newspapers, or radio, or in other media. Still other models make certain assumptions about competitive behaviour, and so on. The point is that all existing approximations of the advertising sales relationship are restricted in application by their assumptions, whether of modest or ambitions proportions. No model exists that applies to any product under any conditions.

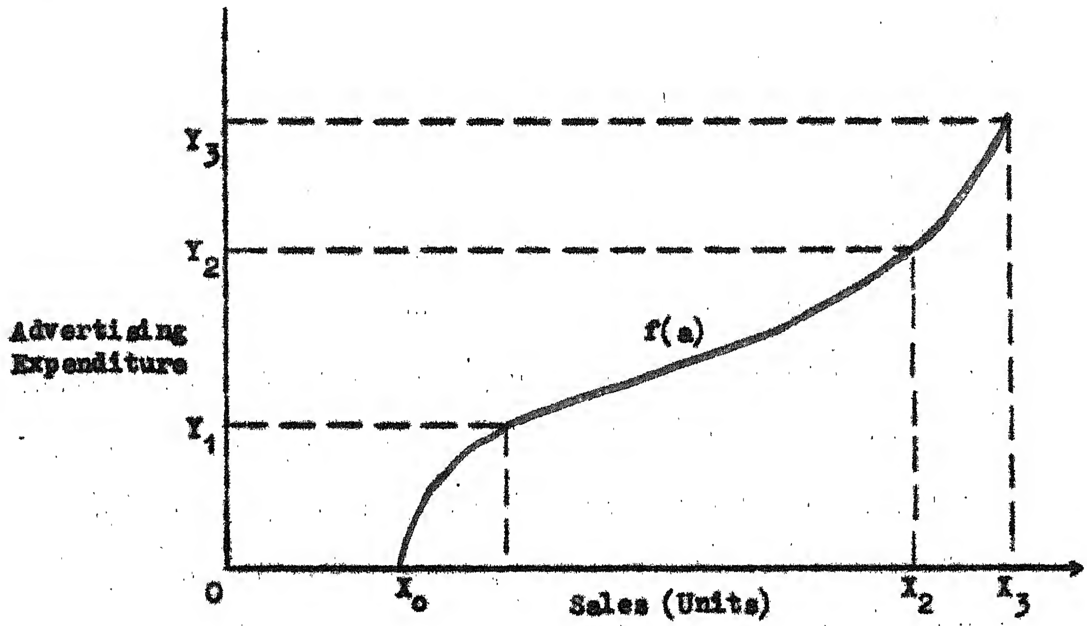
It should be understood that for a model to apply to "any product under any conditions" it need not specify the same sales effect for advertising in every case. Indeed, sales response to advertising would be expected to differ among products and by circumstance. In a universal model, varying conditions could be specified as the parametric values of its variables, as distinguished from limiting models that break down under such varying conditions.

THE EXECUTIVE'S PLIGHT

Given this unfortunate state of affairs, what is the executive to do? Certainly the difficulties - both conceptual and practical - encountered in establishing a workable advertising - sales model in no way diminish the importance of doing so. When huge sums are expended by firms, ostensibly to profitably affect their sales volume, it is essential that the advertising - sales relationship be understood.

The advertising - sales relationship, $f(a)$ is often assumed to look about like that depicted in the accompanying diagram. This diagram shows - as is generally the case - that some sales (X_o units) accrue without any advertising. In the lower expenditure range (less than Y_1) sales are relatively unresponsive, that is, graphically, the percentage change in sales is less than the percentage change in advertising expenditure. In the intermediate expenditure range (between Y_1 and Y_2), however, quite a different relationship holds. In this range, sales are very responsive to advertising, a given percentage change in advertising outlay brining about a far greater than proportional response in sales. Beyond an expenditure of Y_2, the diagram shows that the sales response begins to fall off sharply.

Figure 1 :- The Advertising - sales Relationship



In such a case an expenditure of Y_1 is not enough, in the sense that a few more rupees would increase sales greatly. Expenditure Y_2 is highly effective, but Y_3 is suggestive of waste. Now if f(a) as depicted was known with certainity, and if all relevant costs and unit price were known and quantifiable, an optimum advertising expenditure could be determined, since firms do not know the precise configuration of f(a), however, they must use judgement in specifying the advertising expenditure (Typically, they cannot adequately quantify relevant costs either, but that is another problem.) Very often, they will prefer to err on the high side, at Y_3 or above, to assure that they are at least achieving the desirable level, Y_2. In addition to the direct costs to the firm, this overspending contributes fuel for critics concerned about overcommercialization of our society.[3] Nevertheless, the manager must function now with the best techniques presently available. Thus the problem must be cast in a subjective sense; unless and until some better method is developed, how should the advertising decision be approached?

<u>A LOGICAL SCHEME</u> :-

The answer lies in formulating the advertising program as logically as possible. The following scheme would be logical.

<u>First</u>, the objectives of the advertising program must be formulated, bearing in mind that not all advertising seeks to stimulate sales directly. Indeed, most brand advertising is of an indirect

nature, its purpose is to establish - in conjunction with other promotional methods - a predisposition to buy. Unfortunately such predispositions among consumers do not exhibit a reliable metric dimension. And even if such a state of mind were quantifiable, there remains the problem of ascribing a singular cause (for example, a specific advertising effort) to it. Thus the very purpose of much advertising effort confounds the measurements of its effectiveness.

Typically, then advertising is an integral part of a firms aggregate promotional effort. Initially, therefore, the objectives of the aggregate effort must be resolved and the cost of doing so established. Only then can the role of (and expenditure for) advertising in the promotional mix be assigned. Specifically as regards objectives cannot be overemphasised; the more clearly the purpose of advertising is understood, the more readily its effectiveness can be evaluated.[4]

Objectives should be ordered according to importance as funds may not be available to achieve all of them. The objectives sought through advertising may range from correcting some misconceptions about a product to suggesting new uses or appealing to new classes of customers. In any case, the objectives should lend direction to all the activities in the advertising program, and, at the same time, be consistent with the firm's over - all promotional goals.

Second, the message or messages that will be used to achieve the advertising objectives must be determined. (In practice, this and the following step are usually considered simultaneously). At its core advertising is communications. Thus the firm must communicate with those it hopes to persuade. Simply to communicate, however, is not necessarily to persuade. If advertising messages are to be effective in persuading consumers, they must be designed according to the socioeconomic and sociopsychological traits of those consumers. Simply stated, advertising must "speak the language of the consumer". It is presumed at this stage that the market for the firm's products or services is adequately defined and that appropriate consumer characteristics are known to management. In practice, of course, this knowledge may represent no small investment in research effort.

In addition to knowing how to communicate with would-be consumers, of course, the firm must know what to communicate. That is, it must have a clear cut notion of what ideas its advertising is to impart. This is the reason for defining objectives clearly at the outset. The study of what is communicated by a message is complex, so, for present purposes, it will suffice merely to note that some of the most important variables are the characteristics of the sender, of the receiver, and the influence of the medium of communication, as well as the actual content of the message.[5]

Third, having a message to communicate to consumers presumes that a medium of communication will be available. The third step in the

sequence, then, is to select an advertising medium or media to carry the message to the desired consumers. Typically, many such media will be available, but some, for the purpose at hand, will be better than the rest. The preferable media are those that, because of their peculiar characteristics, are particularly effective in reaching the audience containing the firm's would be customers. Virtually all advertising media make available research results reflecting their audience characteristics. Among the appropriate media, cost comparisons must be made to determine which medium offers the lowest pre-prospect - reached cost. Caution must be exercised at this point to avoid paying for waste audience. Since all established media have existing audiences - only a part of which are likely to be responsible to the firm's advertising - the cost of the medium should be reckoned in terms of only that segment of the total audience of interest to the advertiser.

Cost per - prospect - reached in a useful criterion for media selection only to the extent that the total cost involved is within the budgetary capacity of the firm, of course. For example, although Commercial Radio may exhibit a low relative cost, its high absolute cost precludes it from consideration by small budget advertisers.

Fourth, before the firm commits itself to a particular advertising campangn, the proposal should be tested to assess the likely response of consumers. This may be accomplished through any one of a number of available pretesting techniques, which, in effect, expose a representative sample of consumers to the proposed

advertising or a simulation of it, in order that reaction may be gauged. The idea is to try out the advertising on representative consumers before risking a large expenditure. If the advertising - all or any part of it - is judged ineffective by the jury of consumers in this pretest, the ineffective elements can be changed.

Upto this point, all that can be done to assure effective advertising will have been done. Objectives have been clearly and concisely formulated. With these as a guide, an advertising message has been designed that, in keeping with the characteristics of the consumers, seeks to convey the ideas reflecting those objectives. The most efficient advertising medium has been selected and the proposed advertising scheme has been pretested on a representative sample of consumers. The firm is now ready to go to market with its advertising campaign.

Fifth, because of market dynamics, the imperfections inherent in research techniques, and the vagaries of human nature, however, actual results will seldom conform to the results of the advertising projected on the basis of its pretesting. For this reason, it is necessary that the advertising be evaluated after mass exposure. Management must know the extent to which its advertising did what was expected. This evaluation may be made by the firm or, as is quite common practice, it may be purchased on a syndicated basis from firms specialising in such services. The benefits of evaluation are two-fold : not only does it permit an assessment of

current advertising efforts, but it furnishes a guide for future efforts.

Thus, with the exception of rare cases, the unit of sales that results from a unit of advertising effort cannot be determined accurately. This is because of the fact that advertising functions in an environment comprising many uncontrollable and incalculable factors that may influence the resulting sales.

However, the unit of sales that may result from a unit of advertising effort can be approximately predicted, and this is obviously of enorous consequence for the drawing up of Advertising budgets. The need to measure advertising effectiveness in terms of sales more accurately than has been hitherto possible is very great indeed. Success in this undertaking would permit accurate evaluations of different types of advertisements and advertising themes. Further, it would afford management a reliable method for determining the extent to which advertising is profitable. Thus rules of thumb and witchcraft could be supplanted by scientific evidence as the basis for setting the advertising appropriation. The author disagrees with the view that "under the Marketingconcept a monogenic view of advertising in relation to sales is no longer of importance".[6] A monogenic view, in as much as it pertains to the possibility of predicting and measuring the sales effect of Advertising, is very important, for all the reasons which have been mentioned.

It should be noted that never before have such sophisticated

quantitative techniques and technological tools been at the disposal of the Market Researcher and the Market Analyst, which will certainly aid in building increasingly better models of the Advertising - sales causal process. (Of course, the use of these aids are for too expensive for the smaller companies). For example, the computers stores, extracts, compiles and summarises advertising and sales information. It processes advertising cost data and helps with sales forecasts, Advertising budgets, sales analysis, brand switching and audience target studies. It facilitates the selection of media, advertising themes, and even the pre- and post-testing of the advertisements themselves. It is used in estimating rates of sales decline or increase, saturation limits, sales response constants, and statistical weighting, it successfully manipulates Markov chain matrices, linear programming optima, factor analysis, and other ponderables. There is no doubt that, in the future, the computer will be increasingly and more effectively utilised in constructing a general mathematical model for advertising - sales causality.[7] Again, in the future, such a general mathematical model will doubtless be utilized in testing non-quantitative theories regarding the Advertising process and its communication effectiveness. This kind of theory - testing has already been done in the area of buyer-behaviour - the Howard - Sheth theory of buyer behaviour has been tested by "a union between the computer and multivariate methods."[8] - and will certainly be done in the area of Sales Responsiveness to Advertising.[8A, 8B.]

PART II

REVIEW OF LITERATURE

REVIEW OF LITERATURE

"Much of the unsatisfactory quality of many past studies of the relationship between advertising and sales stems directly from the use of unsatisfactory models and/or data. A common fault is to try to measure current sales as a function of current advertising, as if all advertising were of the 'week-end special' type. This completely overlooks the carry-over effects of advertising. Current sales are better interpreted as a function of the past and current pattern of advertising expenditure.

Another fault is to carry out the investigation as a simple regression instead of a multiple regression. Where historical (non-experimental) data are being used, it is absolutely essential to incorporate measures of the levels of other independent variables during the period, if the net effect of advertising is to be isolated. Still another fault is that most of the investigations employ a single-equation model, thus ignoring some important feed-back relationships between sales and advertising. These relations can only be expressed in a model consisting of a system of simultaneous equations".[9] But multiple regression analysis may not be possible.

The above two paragraphs are a comprehensive summing-up of nearly all the work in this area.

There is general agreement that Advertising effectiveness in terms of sales is probably easiest to measure in mail-order situations. There is disagreement as to when it is hardest to measure. There are authorities who hold the view that the sales effect of Advertising is hardest to measure in mass consumer brand advertising.[10] There are others who hold the view that it is hardest to measure the sales - effectiveness of industrial advertisements, since there is no store-audit service, no consumer panel service; there is lack of accurate data on industrial media audience, and it is also more difficult to pre-test advertisements on a sample of industrial products.[11]

THE WORK OF PALDA AND TELSER

Two studies have been fairly successful in using historical data to identify the sales effect of mass consumer advertising, one by Palda, and the other by Telser.

Paldas' study consisted of a single-equation, multiple regression study of the effect of advertising on the sales of Lydia Pinkham vegetable compound between 1908 and 1960. There were some specific reasons for choosing this product." The firm spent a very high proportion (40 - 60 per cent) of its sales on advertising. Furthermore, it did not employ many of the customary "parameters" of marketing action, sales force, credit, discounts, frequent changes in package, point of purchase efforts, special offerings, etc.

The assumption thus could safely be made that advertising had a measurable effect, on Pinkham's sales. The product itself, Lydia Pinkham's Vegetable compounds, had no close substitute. Competitor's marketing action was not, therefore, a complicating factor to be coped with. By the same token certain allied issues, such as the geographic distribution of Pinkham's marketing effort, could be ignored. During the detailed examination which followed the decision to delve into the Pinkham case, further factors were discovered which added to the simplicity of the ultimate quantitative analysis. On the whole the conclusion was reached that there was a remarkable stability (between 1907 - 1960) in the universe from which the sample observations were obtained".[12]

Palda's major objective was to asse the existence, importance, and measurability of the carry-over (lagged, delayed-action) effects of Pinkham's advertising. He was able to demonstrate that distributed lagged models gave a better fit to the Pinkham data and a better precast than models which did not incorporate lagged effects. He was able to calculate both the short-term and long-term marginal sales effect of advertising. The marginal advertising dollar seemed to increase sales by only $ 0.50 inthe short term, seeming to suggest that Pinkham appropriated too much for advertising. But the long-term marginal sales effect was three times as large; the marginal advertising dollar increased sales by $ 1.63 in the long term.[13] Palda then calculated the post-tax marginal rate of return on the Company's invested advertising dollar. He found it

to be in the neighbourhood of 37 per cent over the whole period, not an implausible figure for a well-established monopolist.

Palda had tested a large number of models for both goodness of fitness and accuracy of prediction. The general model which met both criteria best was one employing Koyck's model of distributed lags. Palda concluded that, "using the distributed lag approach, first the existence and then the relative importance of the operation of lagged advertising effects is all but confirmed in the case of a successful advertiser."

Palda's demand equation, which gave a fairly good fit to the historical sales of Lydia Pinkham's Vegetable compound between the year 1908 and 1960, was as follows[14] :

$$Y = -3649 + .665\,X_1 + 1180 \log X_2 + 774\,X_3 + 32\,X_4 - 2.83\,X_5$$

where,

Y = yearly sales in thousands of dollars.

X_1 = yearly sales (lagged one year) in thousands of dollars.

X_2 = yearly advertising expenditures in thousands of dollars.

X_3 = a dummy variable, taking on the value 1 between 1908-1925 and 0 from 1926 onwards.

X_4 = year (1908 = 0, 1909 = 1, and so on).

X_5 = disposable personal income in billions of current dollars.

The five independent variables on the right helped account for 94 per cent of the yearly variation in the sale of Lydia Pinkham's vegetable compound between 1908 and 1960. To use it as a sales forecasting equation for 1961, it would be necessary to insert figures for the five independent variables. Sales in 1960 should be put in X_1, the log of the company's planned advertising expenditure for 1961 should be put in X_2, 0 should be put in X_3, the numbered year corresponding to 1961 should be put in X_4, and estimated 1961 disposable personal income should be put in X_5. The result of multiplying these numbers by the respective coefficients and summing them gives a sales forecast (Y) for 1961.

Telser study involved cigarettes, another consumer product where disturbing influences tend to be minimal. "Advertising is undountedly the prime mover in the sales of cigarettes. Prices are virtually identical and there is little opportunity for differentiation by other merchandising efforts. The product is quite homogeneous. Seasonal and geographical effects are small. Consumer behaviour and loyalties are fairly stable."[15] Telser reported an elaborate investigation of the relation of sales to advertising for the three largest cigarette brands between 1912 and 1939.[16] In contrast to the simple models used by earlier researchers he employed a Markov probability brand-switching model whose parameters he estimated by least square techniques. Through manipulations of this model, Telser was able to estimate the rate of depreciation of advertising capital and the marginal rate of return on the advertising capital.

He estimated that cigarette advertising outlays built up a fund of good will that depreciated at a rate varying from 15 to 20 percent per year. The marginal rate of return on this capital was about 15 per cent for Lucky Strike and about -6.8 per cent for Camel. He interpreted the negative marginal rate of return on Camel's prewar advertising to indicate that consumers were supplied with more Camel advertising than they wanted.

The conclusions reached by Telser and Palda using only historical, non-experimental, data were made possible by the simplicity of the product environments, the elegance of their respective models, and the satisfactory quality of the data. In the more normal situation the historical effect of advertising on sales would be highly obscured by other variables and the lack of sufficient data.

Quandt has described the statistical hurdles in measuring sales advertising effectiveness from historical data in the normal case. Quandt explored successively the limitations of cross-sectional models, single-equation time series models, and simultaneous equation time series models. He concluded[11]. More than anything, we need to turn back, perhaps to more classical methods of statistical and experimental design. It is possible that the conceptual and practical contamination of data and confounding of models can be avoided by subjecting approximately randomized sets of retail outlets to varying treatments and applying analysis of variance techniques to the results which could then more properly be thought to come from carefully designed experimental situations.[11,17]

Quandt's advice on the potentiality of experimental design for measuring advertising's effect on sales has independently been recognised by an increasing number of advertising researches, who have conducted a number of pioneering advertising experiments with promising results, and a considerable body of experience and theory on the effects of advertising on sales is building up.[18]

However, bearing in mind the limitations of historical studies, let us first review historical studies other than those of Palda and Telser.

OTHER HISTORICAL STUDIES - THE CASE OF PACKER'S TAR SOAP

These studies have tried to determine the existence of a carry-over effect of Advertising on sales. Case histories of Advertising-sales data follow a similar pattern : Company A advertised and experienced high sales; advertising was reduced, at a later date sales declined substantially.[19] However, a reasonably careful investigation of the circumstances of these case histories provides evidence that the reduction in advertising was not the cause of the later sales decline.

Yet such case histories cannot be entirely waved aside and disregarded. From a Bayesian standpoint, they collectively constitute evidence that must be considered in forming a priori judgement concerning the existence of the carry-over effect. The case for the existence of a carry over effect of Advertising on sales is

stronger, given such case histories, than it would be without them.

The case of Packer's Tar Soap is one which often has been cited as an example of a reduction in advertising resulting in a later decline in sales.

In the early 1920s, Packer's Tar Soap was one of the leading solid shampoos on the U.S. market. In the years following World War I Packer's enjoyed particularly large increases in sales of its product which had already sold well for over 60 years. Sales more than doubled between 1917 and 1925.

Beginning in 1926, sales of the Tar Soap started a steady and persistent decline (See Table 1). Several factors could have contributed to these sales losses. One factor was a substantial reduction in advertising made in 1924, followed by further reduction in succeeding years (see Table 2).

An investigation of the circumstances faced by the Company from 1919 through 1929 brings out several possible explanations for the decline in sales of Packer's Tar Soap, the solid shampoo:

1. Shift in preference from bar shampoos to liquid shampoos.
2. Partial boycott by the service wholesalers handling the bar shampoo.
3. Shift in preference from tar shampoos to other kinds of shampoos.
4. Decrease in the relative price of competing shampoos.

TABLE 1

Domestic sales of Packer Products, 1919 - 1929
(Sales in Gross)

Year	Packer's Tar Soap		Packer's Luquid Shampoos		Olive Oil Shampoos	
	Regular size	10 size	Regular size	10 size	Regular size	10 size
1919	15,684	-	1,335	-	-	-
1920	17.562	-	1,872	-	-	-
1921	17,761	-	2,048	-	-	-
1922	18,730	-	2,628	-	-	-
1923	20,356	-	3,188	-	-	-
1924	21,576	-	3,658	-	-	-
1925	21,550	-	4,020	-	-	-
1926	19,408	-	3,383	-	622	-
1927	18,481	-	3,584	-	1,484	-
1928	16,999	1,791	4,123	2,265	1,423	2,012
1929	16,974	1,582	5,285	3,928	2,711	3,659

TABLE 2

Advertising Expenditures for Packer Products, 1919 - 1919

Year	Packer's Tar Soap	Packer's liquid shampoos	Total
1919	$ 91,138	-	$ 91,138
1920	$ 107,395	-	$ 107,395
1921	$ 142,937	-	$ 142,937
1922	$ 155,490	-	$ 155,490
1923	$ 183,520	-	$ 185,520
1924	$ 97,690	$ 65,290	$ 162,980
1925	$ 54,450	$ 114,800	$ 169,250
1926	$ 45,400	$ 118,600	$ 164,000
1927	$ 51,500	$ 115,250	$ 166,750
1928	$ 27,100	$ 113,975	$ 141,075
1929	$ 27,736	$ 183,067	$ 210,803

5. Introduction of Packer's Olive oil shampoo.

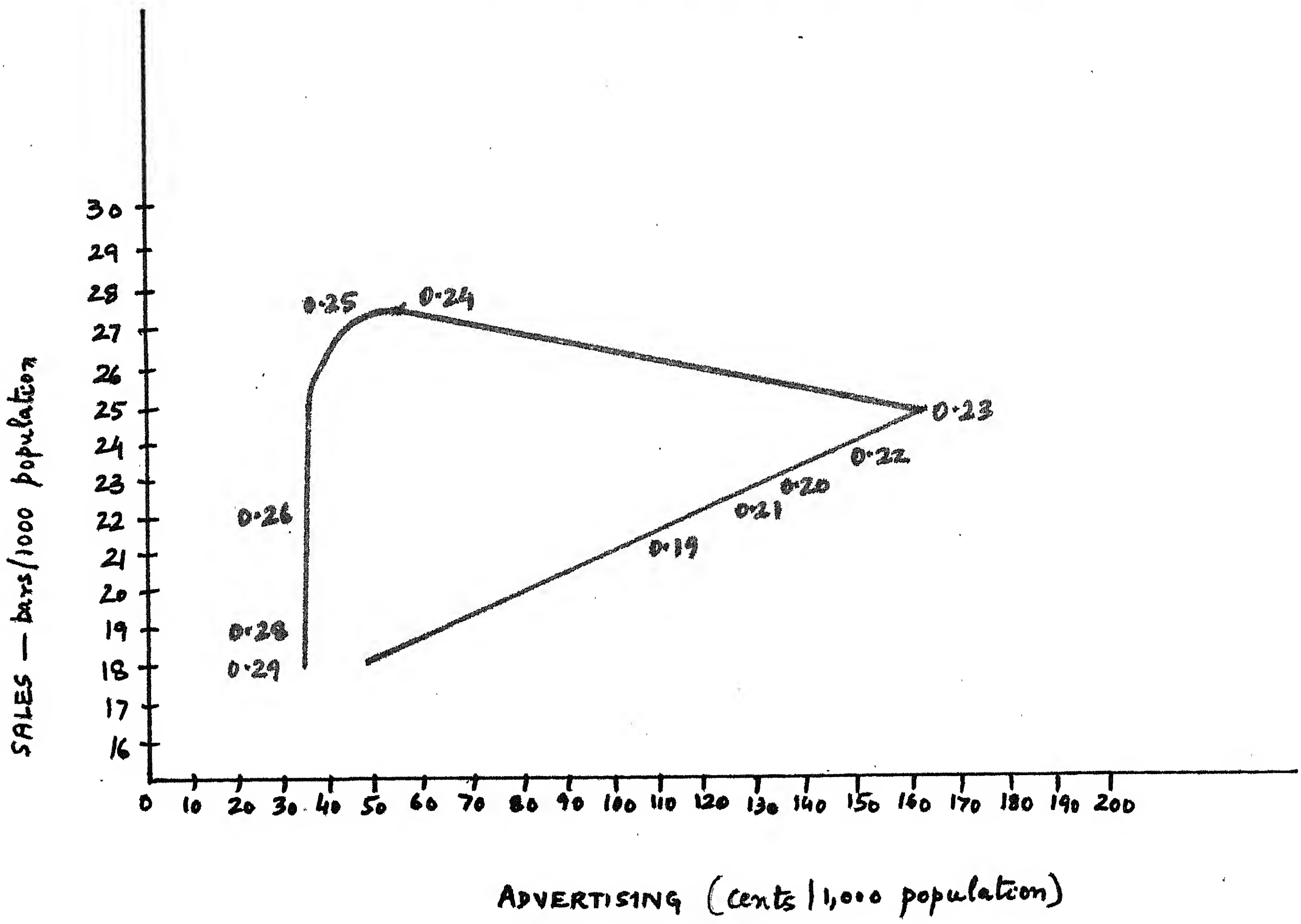
6. Increase in the absolute amount of sales.

7. Reduction in advertising of the Tar soap.

Each of these seven potential reasons for the decline in sales of Packer's tar soap has been examined in detail. The investigation indicated that none of the first six possible explanations were major causal factors.

The relationship of advertising, the seventh factor, and sales of the Tar soap is shown in figure 2.

__Figure 2__ - Advertising and sales of Packer's tar soap, 1919-1929



The evidence with respect to the effect of reduction in advertising is necessarily circumstantial. The most convincing evidence of a causal relationship is provided by comparing sales reductions by geographic area with advertising reduction by geographic area; the two patterns matched closely. On the basis of this and other less direct evidence, the tentative conclusion was drawn that the reductions in advertising initiated in 1924 were causally related to the sales decline starting in 1926.

EXPERIMENTAL STUDIES

Hollander studied the effect of advertising on the sales of an ethical drug. The technique of graphic multivariate correlation was used. While the conclusion was drawn that there was a demonstrable carry-over effect of prior period advertising on later-period sales, the difficulties encountered in the study raised questions about the reliability of the findings. The major problems were in developing suitable indices for the independent variables, and in insuring that these variables were in fact independent. Neither problem was resolved in a fully satisfactory manner.

Nerlove and Waugh carried out a statistical study of the advertising of oranges. Their conclusion was that a lagged effect exists. An interesting finding was that no measurable decay factor could be found for the effect of prior - year advertising on present - year sales for prior periods ranging back as far as ten years.

Scott Paper Company and E.I. du Pont de Nemour Co. have pioneered large-scale experiments for testing the sales-effectiveness of advertising.[20] A test marketing - advertising experiment for "Teflon" coated cookwara was undertaken in the fall of 1962. The conclusion reached was that "there is strong evidence of an advertising 'carry-over effect' from season to season in terms of building market share."

The United States Department of Agriculture studied the effects of advertising on the sales of apples. The conclusion was drawn that "carry-over effects ... did not approach statistical significance".

The Washington State Apple Commission studied the effect of two advertising themes on sales. One theme stressed the various uses of apples (baked apples, fruit combination sales, other dishes), while the other emphasized the healthful quantities of apples (building of strong bodies, dental benefits, etc.). Peter Henderson and his associates set up an elaborate experimental design to find the answer[21]. The experiment involved a total of 72 self-service food stores in six midwestern American cities and ran for 16 weeks. Care was exercised to hold constant or take into account the simultaneous effects of price, competitive advantage, in-store conditions, delayed effects of time, and a number of other extraneous variables. An analysis of the final sales results revealed that the apple use theme was significantly more effective in promoting sales than the health theme.

Vivalde and Wolf, building on studies of advertising effectiveness by Sherman Kingsbury, George E. Kimball and Frank T. Hulsurt, studied the sales response to advertising for seven major industrial concerns. Vivalde and Wold statistically verified a "sales decay constant" that occurred after advertising was reduced or stopped. The Mathematical (theoretical) model developed by Vidale and Wolf will be described later.

THEORETICAL MATHEMATICAL MODELS OF THE SALES RESPONSE TO ADVERTISING

Several models relating sales to advertising over time have been developed. In these models the existence of a carry-over effect has been assumed. Exploring the implications of this assumption on advertising budgeting has been the real purpose.

It is indeed of vital importance to industries of all types and in all nations that a more-or-less "correct" theoretical model interrelating sales and Advertising be built up. "The importance is made apparent by the simple magnitude of advertising expenditures, which annually represents over 2 percent of the gross (U.S.) national product and is expected to approach $ 20 billion per year in 1970."[22] The ramifications and importance of measuring advertising's effect on sales were vividly put by Dr. Alfred N. Watson, Vice-President of Alfred Polits Research, Inc., in a talk before the Operations Research Society of America in 1960. "The solution (measurement of the effect of advertising on sales) would offer

rewards of fabulous proportions to the successful operations researcher and his employer. The possession of a successful formula in the hands of one company could literally spell corporate death for its competitors ..... Perhaps no discovery short of that of a formula for determining future stock prices would have greater impact on American business and the course of our national economy"[23].

THE KUEHN MODEL

The model developed by Alfred A. Kuehn is one of the few models in which the effect of advertising on sales in future time periods is effectively dealt with. "If used with care and discretion, this model may be immediately useful to some companies"[24].

Kuehn assumes that the goal of the firm is the maximisation of profits. His model is based upon empirical relationships describing the brand shifting of households in their purchase of grocery products. These general relationships also seem to apply to markets for some industrial products.[25] The influence of price, product characteristics, and distribution (retail availability and shelf-display space) upon consumer choice of brands are incorporated into the analysis as variables affecting optimal consumer - directed advertise policy. The model does not, however, attempt to evaluate the effect of such advertising upon wholesalers and retailers, or determine optimal policy for the firm with respect

to product quality, pricing, and promotional effort with distributors. The latter are treated within the model as independently determined variables.

The model is developed by maximising the sum of the present and discounted future profits of a brand with respect to its current advertising budget. The sales revenue for the brand is determined by combining empirical brand shifting relationships with assumed, partially tested mechanisms of consumer influence. The resulting equation relates advertising and the effectiveness of price, product characteristics, and distribution for individual brands to changes in their retail sales performance from period to period. Costs are introduced through an advertising cost function and through the assumption that all other costs of production and distribution are a fixed percentage of sales (constant marginal cost apart from advertising). The budgeting model is then optimal in the sense that it maximises the brand's "present worth" given measures or estimates of 11 market, variables and parameters including the brand's gross margin apart from advertising, the level of competitive advertising expenditures, and the competitive strength of the brand with consumers in terms of price, product characteristics, and retail availability.

The implications of Kuehn's model have been examined under conditions of competitive equilibrium and disequilibrium and compared with what has been reported to be business practice. Several profit oriented rules of thumb have been provided by Kuehn's model,

and have been discussed by Kuehn. Kuehn has also advanced suggestions for the use of his model for planning and evaluating advertising budgets.

CONSUMER BRAND SHIFTING - A MARKOV PROCESS

Consumer brand shifting behaviour is analysed. Two characteristics of first-order Markov chains suggest their use as a framework for the analysis of consumer brand shifting behaviour : (1) some simple types of learning behavior can be described as a Markov process, and (2) the first-order Markov chain is mathematically convenient, the equilibrium conditions and the transition states of the system being easily evaluated[26,27,28,29,30,31].

The most direct way in which brand switching can be described as a first-order Markov process is on a purchase to purchase basis. This relationship is not consistent with observed sequences of brands purchased by consumers. The general form of the expression is, however, appropriate for use as a frame for the advertising budgeting problem since a more complex model that does describe consumer brand shifting will be shown to be transformable into an equivalent first order Markov chain.

Table 3 illustrates a first-order purchase to purchase transition probability matrix. The matrix should be interpreted as follows : a consumer who purchased brand A as his last purchase of the product has probability 0.55 of buying A on his next purchase,

TABLE 3

		Brand to be purchased on next buying occasion		
		A	B	C
Brand purchased on last buying occasion	A	.55	.30	.15
	B	.20	.70	.10
	C	.10	.10	.80

0.30 probability of buying brand B and 0.15 probability of buying brand C. If we assume that all consumers buy the product in equal quantities and with equal frequency (e.g., once per time period), the above model will describe shifts in market share for individual brands. For example suppose that the market share for brands A, B and C were 20 percent, 50 per cent, and 30 per cent respectively in the period just past. Then, if the transition probabilities remain constant, brand A share of market in the present period will be 20 percent (.55) + 50 percent (.20) + 30 percent (.10) = 24 per cent. Similarly, brand B will obtain 44 per cent and brand C 32 per cent of the market. This analysis could be repeated to find the shares of market for any number of periods into the future under the assumption that the transition probabilities do not change. Equilibrium market shares could be determined by carrying

out the above process indefinitely (equivalent to taking the transition matrix to a high power) or by solving the three equation system describing equilibrium conditions.

The matrix of transition probabilities might then be interpreted as being the resultant of two sets of consumer-market parameters, a retention factor r_i for each brand i and a merchandising attraction factor a_i where $\Sigma a_i = 1$, all $a_i \geq 0$, and $0 \leq r_i \leq 1$.

The above Markov chain interpretation of consumer brand shifting was found to be very useful. Subsequent empirical analyses of sequences of consumer purchases indicated that purchses of brands by a household prior to the most recent buying occasion have substantial effects upon its choice of a brand when the product is nest purchased. This is inconsistent with the use of the first-order purchase-to-purchase Markov chain which assumes that the probability of a consumer choosing any given brand depends only

TABLE 4

Aggregate Market form of first order Markov Matrix

		Brand to be purchased on next buying occasion		
		A	B	C
Brand purchased on last buying occasion	A	$r_A + (1 - r_A)a_A$	$(1 - r_A)a_B$	$(1 - r_A)a_C$
	B	$(1 - r_B)a_A$	$r_B + (1 - r_B)a_B$	$(1 - r_B)a_C$
	C	$(1 - r_C)a_A$	$(1 - r_C)a_B$	$r_C + (1 - r_C)a_C$

upon the brand last purchased. However, it can be shown that aggregate brand shifting from time period to time period by consumers can be represented as a first-order Markov chain.

An analysis of brand purchase sequences of frozen orange juice by approximately 650 families, members of the "Chicago Tribune" consumer panel during the years 1950 - 1952, has demonstrated that the effect of a consumer's past purchases upon his current choice of a brand decreases exponentially as one goes back into the consumer's purchase history.[32] Thus, each past purchase has an effect equal to some fraction of the effect associated with the purchase following its chronologically. This finding suggested that consumer brand shifting might be described by a stochastic learning model similar to those discussed by Bush and Mosteller.[33]

Experimentation in fitting consumer purchase sequences has demonstrated that the Bush-Mosteller "gain-loss" form of model, generalized to permit incomplete learning and extinction of the brand choice response, describes brand shifting behavior remarkably well. This model has two operators : (1) a gain operator which increases the probability of brand i being purchased in the future, operative when brand i was purchased on the last buying occasion, and (2) a loss operator which reduces the probability of brand i being purchased in the future, operative when some brand other than i was purchased (brand i being rejected) on the last buying occasion. The two operators can be expressed as follows :

$$\text{Gain operator : } P_{i,t} = P_{i,t-1} + g\left(U_i - P_{i,t-1}\right)$$

$$\text{Loss operator : } P_{i,t} = P_{i,t-1} - l\left(P_{i,t-1} - L_i\right) ,$$

where $0 \leqslant L_i \leqslant P_{i,t-1} \leqslant U_i \leqslant 1$; $0 \leqslant g$, $l \leqslant 1$;

and

$P_{i,t}$ = probability of the consumer purchasing brand i on the t^{th} purchase occasion.

g = gain parameter, the fraction of maximum possible gain in purchase probability $(U_i - P_{i,t-1})$ which is realised when the brand is purchased.

l = loss parameter, the fraction of maximum, possible loss in purchase probability $(P_{i,t-1} - L_i)$ which is realised when the brand is rejected. In general, g and l bear subscripts much as do U and L for brand identification. For established brands in homogeneous product-price classes with widespread retail availability, however, these parameters appear to be a constant.

U_i = upper limit of probability of purchase of brand i attained by consumers. $1 - U_i$ is the extent of incomplete adjustment or learning in the limit and, in a closed-market system (i.e. no entry or exit of brands), is equal to $\sum\limits_{j \neq i} L_j$;

L_i = lower limit of probability of purchase of brand i,

approached if a consumer repeatedly rejects the brand. There is incomplete extinction of the purchase response if L_i is greater than 0.

The gain and loss operators can be combined, each weighted by its probability of being operative, to yield the expected value purchase probability relationship

$$P_{i,t} = P_{i,t-1}\left[P_{i,t-1} + g\left(U_i - P_{i,t-1}\right)\right]$$

$$+\left(1 - P_{i,t-1}\right)\left[P_{i,t-1} - l\left(P_{i,t-1} - L_i\right)\right]$$

$$= (1-g)P^2_{i,t-1} + \left(1 - gU_i - l - lL_i\right)P_{i,t-1} + lL_i$$

If the g_i and l_i are each constant for all brands as discussed above, then, in a closed-market system (no entry or exit of brands) g also equals l and we obtain

$$P_{i,t} = P_{i,t-1}\left(1 - g\left[1 - U_i + L_i\right]\right) + gL_i .$$

If we now define the states of the system to be purchase probabilities rather than the brand last purchased, the above relationships can be treated as a first-order Markov process. The parameters and the most recent state of the system determine completely the expected probability of purchase on the current buying occasion. To illustrate the equivalence of this aggregate

form of the Bush-Mosteller learning model with the aggregate market form of the first-order Markov purchase to purchase matrix (Table 4), let us compare equilibrium share of market for brand i and the rate at which the market approaches equilibrium (Table 5).

TABLE 5

Comparison of First Order Markov process and the Bush-Mosteller learning Model

	First-Order Markov	Bush-Mostellar Model
Equilibrium	$\dfrac{a_i/(1-r_i)}{\Sigma[a_i/(1-r_i)]}$	$\dfrac{L_i}{\Sigma L_i}$
Rate of approach to Equilibrium	$a_i(1-r_j)+(1-a_i)(1-r_i)$	$g\,\Sigma L_i$

The equilibrium and rate of approach to equilibrium relationships shown above for the Bush-Mosteller model assume a closed-market system in which $g = 1$ and $U_i = 1 - \Sigma L_j (j \neq i)$. No simple expression for the rate of approach to equilibrium is possible for the first-order Markov process if there are more than two r's. If $r_i = r_j = r$, this expression simplifies to $(1-r)$ since $a_i + a_j = 1$.

Thus, the Bush-Mosteller model with n+1 independent parameters $(L_1, \ldots\ldots L_n, g)$ can be transformed into an equivalent simple Markov chain with n+1 independent parameters $(a_1, \ldots\ldots a_n, r)$ for

the purpose of describing shifts in brand market shares. Each of these models can be extended to provide a better description of data developed from nonhomogeneous markets (markets with significant differences among brands in terms of retail availability, products characteristics, or price) by adding two independent parameters to describe competition (g_2, r_2) or, preferably, $2(n-1)$ parameters ($g_2, \ldots\ldots, g_n, r_2, \ldots\ldots r_n$), one g and one r for each compbtitor. Estimates of equilibrium conditions provided by the two models using data accumulated over several average purchase cycles of the product class are roughly comparable. Estimates of the rate of approach to equilibrium differ widely, however. In a study of frozen orange juice, for example, the Markov purchase-to-purchase matrix yielded an approach rate of 0.5 (50 per cent of the deviation from equilibrium per purchase occasion) whereas the Bush-Mosteller model indicated a rate of less than 0.05. The learning model gives recognition to the fact that much of the observed shifting between brands of most products is not indicative of a major shift inthe purchasing habits of the consumer. The Bush-Mosteller type analysis is, for example, particularly appropriate to the study of products in which several brands are used simultaneously by a family as a result of a desire for variety (ready-to-eat cereals), individual family-member preferences (toothpaste), and distinct end uses (bar soap). Similarly, consumers who frequently shift their patronage among stores will tend to be brand shifters insofar as there are differences in availability of individual brands.

It is likely that the first-order Markov chain will prove to be of primary usefulness since it is more easily manipulated and, in the majority of cases, appears to provide an adequate interpretation of market conditions when reasonable care in its use is exercised. More depth of analysis is required with the Bush-Mosteller model both to establish its advantages, if any, over the simple Markov chain for the evaluation of marketing strategy and tactics, and to provide improved interpretation and methods of application for the latter model. One method of modifying the purchase-to-purchase Markov chain to more closely approximate the results provided by the Bush-Mosteller model is to represent within the transition matrix the shifting mix of brands for individual consumers from one time period to a subsequent time period. These might be consecutive periods of time or, in the case of brands influenced by seasonal factors, corresponding periods in the annual cycle. The longer the time periods used, the smaller the effect of short-term brand shifting and the closer the agreement between this estimate of the rate of approach to equilibrium and the Bush-Mosteller estimate. As the time period becomes longer, however, greater time lags are experienced in identifying significant long-term shifts in the market position of brands. Some balance of these two considerations is needed. Products purchased with high frequency might be studied on as short as a month-to-month basis. Products purchased with low frequency and products of a seasonal nature on the other hand might be studied using time period (or lags) of up to twelve months.

KUEHN'S GENERAL EXPRESSION RELATING SALES AND ADVERTISING :

We have seen that the general form of the Bush-Mosteller stochastic learning model can be used to describe and simulate brand shifting by customers. Also, this model produces aggregate market trends over time among brands which can be described by an equivalent first-order Markov matrix. Using the Markov model with some changes in definition of variables we obtain the following difference equation as a description of aggregate brand shifting in the market :

$$S_{i,t} = r_i S_{i,t-1} + I_{t-1}(1 - \bar{r}_t) Z_{i,T} ,$$

where

$S_{i,t}$ = unit sales of brand i in time period t,

r_i = repeat probability of purchase for brand i (1 minus the rate of decay of brand loyalty), assumed to be constant over time,

I_{t-1} = $\Sigma S_{i,t-1}$, the unit sales of all brands (industry sales) in time period t-1, assumed to be constant over time. (This restriction is relaxed later on).

$\bar{r}_t$ = weighted average of decay rate of brand loyalty for the industry, not constant over time (unless the r_i's are all equal) because of shifts in relative sales volume of individual brands, i.e.,

$$\frac{\sum_i r_{i,t} S_{i,t-1}}{\sum_i S_{i,t-1}}$$

$Z_{i,T}$ = fraction of "potential brand shifters" attracted to brand i in time period t as a result of the brand's retail price and availability relative to competition at time t and the advertising in its behalf during time period T. The distinction between t and T takes cognizance of the fact that some time lag may exist between the advertising expenditures and the sales results.

Next, consider an expression for Z_i

$$Z_i = b_p \frac{P_i}{\Sigma P_i} + b_d \frac{D_i}{\Sigma D_i} + b_a \frac{A_i}{\Sigma A_i} + b_{pd} \frac{(PD)_i}{\Sigma(PD)_i} + b_{pa} \frac{(PA)_i}{\Sigma(PA)_i}$$

$$+ b_{da} \frac{(DA)_i}{\Sigma(DA)_i} + b_{pda} \frac{(PDA)_i}{\Sigma(PDA)_i} ,$$

where

$$\Sigma b_k = 1, \quad \Sigma P_i = \Sigma D_i = \Sigma A_i = 1 .$$

$$(PD)_i = P_i D_i; \quad (PA)_i = P_i A_i ; \quad (DA)_i = D_i A_i$$

$$(PDA)_i = P_i D_i A_i$$

P_i = the share of brand shifters which would be attracted by brand i if product characteristics and price were the only merchandising variables (i.e., $b_p = 1$, b_d, b_a, ... $b_{pda} = 0$). P_i is also the equilibrium share of market if it is constant over time, $b_p = 1$, and all the r_i's are equal. Similarly, D_i and A_i represent the share of potential brand shifters that would be attracted if

distribution - display space and advertising, respectively, were the only influential merchandising variables.

$\frac{(PD)_i}{\Sigma(PD)_i}$ = the share of brand shifters which would be attracted by brand i if the interaction (joint) effect of product characteristics - price and distribution - display were the only influence upon consumer choice. The other interaction effects are defined in similar fashion.

The expression for Z_i is quite general in that it incorporates all of the direct and interaction effects of the three variables P_i, D_i and A_i. The choice of these variables was based on two considerations (1) their importance in influencing consumer brand choice, and (2) methods which might be used to measure merchandising variables. Some research results show promise in developing measures of P_i and D_i.

The expression for Z_i can be simplified by certain assumptions :

1. There must be some retail distribution and display for a brand if it is to achieve any sales (i.e., if $D_i = 0$, then $Z_i = 0$).

2. There must be some attraction associated with the brand's retail price and the consumer's perception of its product characteristics if it is to achieve any sales (i.e., if $P_i = 0$, then $Z_i = 0$).

The first assumption requires b_p, b_a and b_{pa} to be equal to 0.

The second assumption sets b_d, b_a, and b_{da} equal to 0. Consequently, if we normalize the $(PD)_i$ such that $\Sigma(PD)_i = 1$, the expression for Z_i can be simplified to

$$Z_i = b_{pd}(PD)_i + b_{pda} \frac{(PDA)_i}{\Sigma(PDA)_i} .$$

Hence,

$$S_{i,t} = r_i S_{i,t-1} + I_{t-1}(1 - \bar{r}_t)\, b_{pd}(PD)_i + I_{t-1}(1 - \bar{r}_t)\, b_{pda} \frac{(PDA)_i}{\Sigma(PDA)_i} .$$

The term $I_{t-1}(1 - \bar{r}_t)$ represents the total industry sales to "potential brand shifters". Not all of these sales will actually result in brand shifting since the customer may be drawn back to his old brand by its merchandising activities, hence the term potential. Consumer entry into and exit from the market for the product is not included in the above equation. If we assume a constant and equal rate of exit for customers of each brand and, furthermore, assume that entries (newcomers) are distributed among brands as are the potential brand shifters, we obtain

$$S_{i,t} = r_i e S_{i,t-1} + I_{t-1}\left[g + (1 - \bar{r}_t)\, e \right] b_{pd}(PD)_i + I_{t-1}\left[g + (1 - \bar{r}_t)\, e \right] b_{pda} \frac{\Sigma(PDA)_i}{\Sigma(PDA)_i} ,$$

where,

e = probability of survival of past customers of the product, i.e., (1 - probability of exit), and

g = entry of potential customers to the market for the product as a fraction of the size of the market in the previous time period.

Thus, the sales volume of brand i during time period t is made up of three parts : (1) the surviving customers of the previous period who remain loyal to the brand, (2) the newcomers and surviving potential brand shifters attracted to or retained by the brand as a result of the brand's product characteristics, price and retail availability $(PD)_i$, and (3) the newcomers and surviving potential brand shifters attracted to or retained by the brand as a result of the brand's total merchandising activities, including advertising $(PDA)_i$. Let $k = e + g$, the net growth of the industry sales per period, then $I_t = kI_{t-1}$ and $I_{t-1} = k^{t-1} I_o$, where I_o = industry sales in an arbitrary time period $t = 0$. Substituting in the expression for $S_{i,t}$, we have

$$S_{i,t} = r_i e S_{i,t-1} + I_o k^{t-1}(k - \bar{r}_t e)\, b_{pd}(PD)_i + I_o k^{t-1}(k - \bar{r}_t e)\, b_{pda} \frac{(PDA)_i}{\Sigma(PDA)_i} .$$

Since $\Sigma(PDA)_i = \Sigma(PD)_i \; A_i$, we have

$$\frac{(PDA)_i}{\Sigma(PDA)} = \frac{(PD)_i \, A_i}{\Sigma(PD) \, A} = \frac{(PD)_i \, A_i}{(PD)_i \, A_i + (PD)_C (1 - A_i)}$$

$$= \frac{(PD)_i\, A_i}{(PD)_C + (PD)_i\, A_i - (PD)_i\, A_i} \;,$$

where :

$$(PD)_{c,t} = \frac{\sum_{j \neq i} (PD)_{j,t}\, A_{j,T}}{\sum_{j \neq i} A_{j,T}} \;, \quad \text{and} \quad \sum_{j \neq i} A_{j,T} = 1 - A_{i,T} \;.$$

Substituting the expression for $\frac{(PDA)_i}{\Sigma(PDA)_i}$ in the expression for $S_{i,t}$, we have

$$S_{i,t} = r_i e S_{i,t-1} + I_o k^{t-1}(k - \bar{r}_t e)(PD)_i \left[b_{pd} + \frac{b_{pda}\, A_{iT}}{(PD)_c + (PD)_i A_{iT} - (PD)_c\, A_{iT}} \right]$$

In the above expression, $T = t - L$, where L represents the time lag between the advertising expenditures and its influence upon new and shifting customers. For the $L+1$ periods from $t = 0$ to $t = L$, sales are already determined by customer loyalties, the influence of product characteristics, price and retail availability, and such advertising as might have been done in periods $-L$ through 0. For convenience, therefore, we can set the advertising variable $A_{i,T} = 0$ for $T = t - L \leq 0$ since sales during the first L periods do not affect the optimising decision. Then we have,

<u>for $t \leq L$</u>,

$$S_{i,t} = r_i e S_{i,t-1} + I_o k^{t-1}\left(k - \bar{r}_t e\right) b_{pd}(PD)_i .$$

At $t = 1$, this expression is

$$S_{i,1} = r_i e S_{i,o} + I_o k^o \left(k - \bar{r}_1 e\right) b_{pd}(PD)_i .$$

At $t = 2$, this expression is

$$S_{i,2} = r_i e S_{i,1} + I_o k' \left(k - \bar{r}_2 e\right) b_{pd}(PD)_i ,$$

which can be expressed in terms of $S_{i,o}$ as

$$S_{i,2} = (r_i e)^2 S_{i,o} + (r_i e)(I_o k^o b_{pd})(PD)_i(k - \bar{r}_1 e)$$

$$+ (I_o k' b_{pd})(PD)_i(k - \bar{r}_2 e) .$$

At $t = 3$, the expression is

$$S_{i,3} = r_i e S_{i,2} + I_o k^2 \left(k - \bar{r}_3 e\right) b_{pd}(PD)_i$$

which can be expressed as

$$S_{i,3} = (r_i e)^3 S_{i,o} + (r_i e)^2 (I_o k^o b_{pd})(PD)_i(k - \bar{r}_1 e)$$

$$+ (r_i e)^1 (I_o k^1 b_{pd})(PD)_i(k - \bar{r}_2 e)$$

$$+ (r_i e)^o (I_o k^2 b_{pd})(PD)_i(k - \bar{r}_3 e)$$

Therefore, the general form for $t \leq L$ is

$$S_{i,t} = (r_i e)^t S_{i,o} + \sum_{T=o}^{t-1} (r_i e)^{t-T-1} I_o k^T b_{pd}(PD)_i (k - \bar{r}_{T+1} e)$$

<u>For $t > L$</u> : For $t = L + 1$, we have

$$S_{i,L+1} = r_i e S_{i,L} + I_o k^L b_{pd}(PD)_i (k - \bar{r}_{L+1} e)$$

$$+ I_o k^L b_{pda}\left(k - \bar{r}_{L+1} e\right) \frac{(PD)_i A_{i,1}}{(PD)_o + [(PD)_i - (PD)_o] A_{i,1}}$$

Putting $t = L$ in the general form of expression for $S_{i,t}$, we have

$$S_{i,L+1} = (r_i e)^{L+1} S_{i,o} + (r_i e) \sum_{T=o}^{L-1} (r_i e)^{L-T-1} I_o k^T b_{pd}(PD)_i (k - \bar{r}_{T+1} e)$$

$$+ I_o k^L b_{pd}(PD)_i (k - \bar{r}_{L+1} e)$$

$$+ I_o k^L b_{pda}\left(k - \bar{r}_{L+1} e\right) \frac{(PD)_i A_{i,1}}{(PD)_o + [(PD)_i - (PD)_o] A_{i,1}} .$$

Combining the second and third terms,

$$S_{i,L+1} = (r_i e)^{L+1} S_{i,o} + I_o b_{pd} (PD)_i \sum_{T=o}^{L} (r_i e)^{t-T-1} k^T \left(k - \bar{r}_{T+1} e\right)$$

$$+ I_o b_{pda} k^L \left(k - \bar{r}_{L+1} e\right) \frac{(PD)_i A_{i,1}}{(PD)_o + [(PD)_i - (PD)_o] A_{i,1}} .$$

At $t = L+2$,

$$S_{i,L+2} = (r_i e) S_{i,L+1} + I_o k^{L+1} b_{pd} (PD)_i \left(k - \bar{r}_{L+2} e\right)$$

$$+ I_o k^{L+1} b_{pda} \left(k - \bar{r}_{L+2} e\right) \frac{(PD)_i A_{i,2}}{(PD)_o + [(PD)_i - (PD)_o] A_{i,2}} ,$$

so that by substitution for $S_{i,L+1}$ previously calculated, we get after simplification

$$S_{i,L+2} = (r_i e)^{L+2} S_{i,o} + I_o b_{pd} (PD)_i \sum_{T=o}^{L+1} (r_i e)^{t-T-1} k^T \left(k - \bar{r}_{T+1} e\right)$$

$$+ I_o b_{pda} \sum_{T=1}^{t-L} (r_i e)^{t-L-T} k^{L+T-1} \left(k - \bar{r}_{L+T} e\right)$$

$$\frac{(PD)_i A_{i,T}}{(PD)_o + [(PD)_i - (PD)_o] A_{i,T}} .$$

Continuing, <u>the general expression relating sales and advertising</u> becomes

$$S_{i,t} = (r_i e)^t S_{i,o} + I_o b_{pd} (PD)_i \sum_{T=o}^{t-1} (r_i e)^{t-T-1} k^T \left(k - \bar{r}_{T+1} e\right)$$

$$+ I_o b_{pda} \sum_{T=1}^{t-1} (r_i e)^{t-L-T} k^{L+T-1} \left(k - \bar{r}_{L+T} e\right)$$

$$\frac{(PD)_i A_{i,T}}{(PD)_c + [(PD)_i - (PD)_c] A_{i,T}} .$$

This General Expression holds for all time periods (t).

<u>INFERENCES FROM KUEHN'S GENERAL EXPRESSION RELATING SALES TO ADVERTISING</u>

<u>For $t < L$</u>, the firm is <u>not</u> in a position to influence sales by means of advertising, since, for $t < L$, advertising does not influence consumer behavior. Sales during these L periods will consist of demand from (1) loyal customers held over from $t = 0$, (2) potential brand shifters attracted by the firm's product, price, and/or distribution, and (3) potential brand shifters attracted by the interaction of these same variables with advertising during periods $1 - L \leq T \leq 0$. (For simplicity, Kuehn has assumed that no advertising was done during these periods, an assumption which is no way affects the optimising of advertising

budgets during time periods when $t > 0$).

For $t \geq L+1$ it is possible, through advertising at time $t-L$, to increase customer response. Such increases will also augment sales in subsequent periods because of the habitual behaviour of consumers.

UTILITY OF KUEHN'S GENERAL EXPRESSION RELATING SALES AND ADVERTISING IN DERIVING OTHER IMPORTANT EXPRESSIONS

The General Expression can be taken as a basis for developing an advertising decision Rule (an optimum advertising budgetting model), in determining advertising expenditures at competitive equilibrium, in determining the share of market at competitive equilibrium, in arriving at a ratio of profits at competitive equilibrium. The optimum advertising budgetting model has wide application to Advertising Research.

MAXIMISING PROFITS WITH RESPECT TO ADVERTISING

Management should aim to maximise the discounted profits of the firm over time, beginning with time period $t = L+1$. Consequently, summing over all time periods from $t = L+1$ to $t = \infty$ the difference between the profit margins apart from advertising and the corresponding advertising expenditures, each discounted to $t = 0$, we obtain for brand i the profit function,

$$\pi_i = \sum_{t=L+1}^{\infty} \left(\rho^t m_i S_{i,t} - \rho^T C_{i,T} \right)$$

which is equivalent to

$$\pi_i = \sum_{t=L+1}^{\infty} \rho^t m_i S_{i,t} - \sum_{T=1}^{\infty} \rho^T C_{i,T} ,$$

where,

$$\rho = \text{discount factor} = \frac{1}{1 + \text{rate of interest}}$$

m_i = profit margin per unit sale apart from advertising costs

$C_{i,T}$ = advertising expenditures for brand i at time T.

Before we can maximize the profit function π_i, we must specify an advertising cost function.

In Kuehn's General Expression relating sales and Advertising,, the term $A_{i,T}$ was used to represent the influence of the advertising in behalf of brand i relative to that of competition. However, the cost of obtaining such an advertising impact was not specified. Now, "Advertising Effectiveness" is defined such as to satisfy the advertising cost function

$$\frac{A_{i,T}}{C_{i,T}E_{i,T}} = \frac{A_{j,T}}{C_{j,T}E_{j,T}} = \frac{\sum_{j \neq i} A_{j,T}}{\sum_{j \neq i} C_{j,T} E_{j,T}} ,$$

where,

$C_{i,T}$ = advertising expenditure in behalf of brand i during period T.

$E_{i,T}$ = Advertising Effectiveness in behalf of brand i during period T.

Let $\sum_{j \neq i}' C_{j,T} E_{j,T} = C_{o,T} E_{o,T}$. Since $\sum_{i}' A_{i,T} = 1$,

the Advertising cost function for brand i at time T,

$$C_{i,T} = C_{o,T} \frac{E_{o,T}}{E_{i,T}} \frac{A_{i,T}}{1 - A_{i,T}} ,$$

where

$C_{o,T} = \sum_{j \neq i}' C_{j,T}$, total advertising expenditures by competitors at time T

$E_{o,T} = \dfrac{\sum_{j \neq i} C_{j,T} E_{j,T}}{\sum_{j \neq i} C_{j,T}}$, the average weighted effectivenss of advertising in behalf of competitive brands in time period T.

It is now possible to determine optimal advertising expenditures in the framework of the model in each time period from T = 1 until T = ∞ on the basis of the profit function π_i and the advertising cost function $C_{i,T}$. The advertising expenditures obtained for T = 2 to T = ∞ are optimal, given present information, i.e., the expected values of the noncontrolled variables in all subsequent time periods. In practice, we would only determine the advertising

expenditure for the first period at the end of which time we would compute the expenditure for the second period on the basis of the information then available, and so forth. Under certainity, this decision rule is optimal. Given uncertainity, this procedure would be optimal in the sense of maximising expectation of profits if profit were a quadratic function of $A_{i,T}$. These conditions would be approximated if the changes in $A_{i,T}$ from period to period were small, that is, no major readjustments from period to period due to error in expectations

To find the optimum expenditure in period T, we take the partial derivative of the profit function with respect to $C_{i,T}$.

$$\frac{\partial \pi_i}{\partial C_{i,T}} = \frac{\partial \sum_{t=L+1}^{\infty} m_i \rho^t S_{i,t}}{\partial C_{i,T}} - \frac{\partial \sum_{T=1}^{\infty} \rho^T C_{i,T}}{\partial C_{i,T}}$$

The profit function being continuous, $\dfrac{\partial \pi_i}{\partial C_{i,T}} = \dfrac{\partial \pi_i}{\partial A_{i,T}} \left[\dfrac{dA_{i,T}}{dC_{i,T}} \right]$

Calculations show that $\dfrac{A_{i,T}}{C_{i,T}} = \dfrac{E_{i,T}(1 - A_{i,T})^2}{C_{o,T}\, E_{o,T}}$, and $\dfrac{\partial \pi_i}{\partial A_{i,T}}$

$$= \frac{m_i I_o b_{pda}(k - \bar{r}_{L+T}\, e)(\rho k)^{L+T}}{(1 - r_i e)\, k} \times \frac{(PD)_i\,(PD)_o}{[(PD)_o + [(PD)_i - (PD)_o]\, A_{i,T}]^2}$$

$$- \frac{\rho^T C_{o,T} E_{o,T}}{E_{i,T}(1 - A_{i,T})^2} .$$

Hence,

$$\frac{\partial \pi_i}{\partial C_{i,T}} = \Bigg[\frac{m_o\, I_o\, b_{pda}\,(k - \bar{r}_{L+T}\, e)\,(\rho k)^{L+T}}{(1 - \rho r_i\, e)\, k}$$

$$\times \frac{(PD)_i\,(PD)_c}{[(PD)_c + [(PD)_i - (PD)_c\,]\,A_{i,T}]^2} \cdot \frac{E_{i,T}\,(1 - A_{i,T})^2}{C_{c,T}\,E_{c,T}} \Bigg] - \rho^T \;.$$

Putting $\frac{\partial \pi_i}{\partial C_{i,T}} = 0$, and simplifying,

$$\frac{(PD)_c}{(PD)_i} + \frac{2A_{i,T}}{1 - A_{i,T}} + \frac{(PD)_i\,A_{i,T}^2}{(PD)_c\,(1 - A_{i,T})^2}$$

$$= \frac{m_i\, I_o\, E_{i,T}\, b_{pda}\,(k - \bar{r}_{L+T}\, e)\,(\rho k)^L\, k^{T-1}}{C_{c,T}\, E_{c,T}\,(1 - \rho\, r_i\, e)} \;.$$

Let

$(PD)_R = \frac{(PD)_i}{(PD)_c}$, the ratio of the merchandising strength of brand i, apart from advertising, to that of its competitors.

$E_R = \frac{E_i}{E_c}$, the effectiveness of advertising in behalf of brand i relative to that of competing brands.

Replacing $A_{i,T}$ terms by $C_{i,T}$, we obtain the <u>Optimal Advertising Decision Rule</u> (<u>Opt. Advertising Budgetting Model</u>)

$$C_{i,T} = C_{c,T}\left[\sqrt{\frac{m_i\, I_o\, b_{pda}(k - \bar{r}_{L+T}\, e)\,(\rho k)^L\, k^{T-1}}{C_{c,T}(1 - \rho r_i\, e)\, E_R(PD)_R}} - \frac{1}{E_R(PD)_R}\right] .$$

ADVERTISING EXPENDITURES AT COMPETITIVE EQUILIBRIUM

Kuehn has defined "competitive equilibrium" as that state of the market at which no brand or firm can improve its profitability by modifying its advertising expenditures.

Setting $C_{c,T}$ equal to the optimal advertising expenditure for brand j, the optimal advertising expenditure for brand j is

$$C_{j,T} = C_{i,T}\left[\sqrt{\frac{m_j\, I_o\, b_{pda}(k - \bar{r}_{L+T}\, e)\,(\rho k)^L\, k^{T-1}\, E_R(PD)_R}{C_{i,T}(1 - \rho r_j\, e)}} - E_R(PD)_R\right]$$

Let $W = I_o\, b_{pda}(k - \bar{r}_{L+T}\, e)\,(\rho k)^L\, k^{T-1}$. Then, putting the value of $C_{j,T}$ in the optimal decision rule,

$$C_{i,\text{equilibrium}} = \frac{W\, m_i\, m_R\, E_R(PD)_R}{(1 - r_j\, e)\left[\frac{(1 - \rho r_i\, e)}{(1 - r_j\, e)} + m_R\, E_R(PD)_R\right]^2} .$$

To further simplify this relation, define $Q_R = m_R E_R(PD)_R$ and $\frac{1 - \rho r_i\, e}{1 - \rho r_j\, e} = (1 - \rho r\, e)_R$. Then,

$$C_{i,\text{equilibrium}} = \frac{W m_i Q_R}{(1-\rho r_j e)\left[(1-\rho r e)_R + Q_R\right]^2} .$$

For brand j,

$$C_{j,\text{equilibrium}} = \frac{W m_j (1/Q_R)}{(1-\rho r_i e)\left[1/(1-\rho r e)_R + 1/Q_R\right]^2}$$

Multiplying Numerator and denominator by $Q_R^2(1-\rho r e)_R^2$;

$$C_{j,\text{equilibrium}} = \frac{W m_j Q_R (1-\rho r e)_R^2}{(1-\rho r_i e)\left[(1-\rho r e)_R + Q_R\right]^2}$$

and total Industry Advertising expenditures are

$$C_{i,eq} + C_{j,eq} = \frac{W Q_R}{\left[(1-\rho r e)_R + Q_R\right]^2}\left[\frac{m_i}{(1-\rho r_j e)} + \frac{m_j(1-\rho r e)_R^2}{(1-\rho r_i e)}\right]$$

The ratio of advertising expenditures for brand i relative to brand j is (if advertising budget C_j is set at x (100 /) of its equilibrium value and firm i responds so as to maximise its profits),

$$\frac{C_{i,\text{optimal}}}{C_{j,100x\,/\,eq}} = \frac{m_R (x)^{-1/2}}{(1-\rho r e)_R} - \frac{1-(x)^{-1/2}}{E_R (PD)_R} .$$

Therefore,

$$\frac{C_{i,eq}}{C_{j,eq}} = C_R = \frac{m_i(1 - \rho r_j e)}{m_j(1 - \rho r_i e)} = \frac{m_R}{(1 - \rho r e)_R}$$

SHARE OF MARKET AT COMPETITIVE EQUILIBRIUM

Dividing both sides of expression for $S_{i,t}$ by $I_o k^t$ and putting

$$\frac{S_{i,t}}{I_o k^t} = \frac{S_{i,t-1}}{I_o k^{t-1}} ,$$

we have

$$\frac{S_{i,t}}{I_o k^t} = \frac{(k - \bar{r}_t e)}{(k - r_i e)} \left[b_{pd}(PD)_i + \frac{b_{pds}(PD)_i A_{i,T}}{(PD)_j + [(PD)_i - (PD)_j] A_{i,T}} \right]$$

Since $(PD)_i + (PD)_j = 1$ and $\frac{(PD)_i}{(PD)_c} = (PD)_R$ [where $(PD)_c$ in the two-brand case equals $(PD)_j$], dividing the numerator and the denominator by $(PD)_j$ gives

$$\frac{S_{i,t}}{I_o k^t} = \frac{k - \bar{r}_t e}{k - r_i e} \left[\frac{b_{pd}(PD)_R}{(PD)_R + 1} + \frac{b_{pds} A_{i,T}(PD)_R}{1 - A_{i,T}[1 - (PD)_R]} \right]$$

Since $\frac{A_{i,T}}{(1 - A_{i,T})} = \frac{C_i E_i}{C_c E_c} = C_R E_R$, and $C_R = \frac{m_R}{(1 - \rho r e)_R}$ at competitive equilibrium, dividing the numerator and denominator by $(1 - A_{i,T})$, and putting $m_R\, E_R(PD)_R = Q_R$,

$$\frac{S_{i,t}}{I_o k^t} = \frac{k - \bar{r}_t e}{k - r_i e} \left[\frac{b_{pd}(PD)_R}{1 + (PD)_R} + \frac{b_{pda} Q_R}{(1 - \rho r e)_R + Q_R} \right]$$

For brand j,

$$\frac{S_{j,t}}{I_o k^t} = \frac{k - \bar{r}_j e}{k - r_j e} \left[\frac{b_{pd}}{1 + (PD)_R} + \frac{b_{pda}(1 - \rho r e)_R}{(1 - \rho r e)_R + Q_R} \right] .$$

Then,

$$S_R = \frac{S_{i,eq}}{S_{j,eq}}$$

$$= \frac{1}{(k - re)_R} \left[\frac{b_{pd}[(PD)_R Y_R + (PD)_R Q_R] + b_{pda}[Q_R + (PD)_R Q_R]}{b_{pd}[Y_R + Q_R] + b_{pda}[Y_R + Y_R (PD)_R]} \right]$$

where $Y_R = (1 - \rho r e)_R$. This gives the equilibrium sales and share of market ratio between brands i and j in this two-brand case (and between any pair of brands in multibrand markets).

RATIO OF PROFITS AT COMPETITIVE EQUILIBRIUM

The equilibrium profit ratio between brands i and j in a two-brand market in time period t can be represented by

$$\pi_{R,t,eq} = \frac{m_i S_{i,t,eq} - C_{i,T+L,eq}}{m_j S_{j,t,eq} - C_{j,T+L,eq}}$$

The profit for brand i in the two-brand-market in time period t is, therefore,

$$\pi_{i,t} = m_i S_{i,t,eq} - C_{i,T+L,eq}$$

$$= m_i I_o k^t \, \frac{k - \bar{r}_t e}{k - r_i e} \left[b_{pd}(PD)_i + \frac{b_{pda}\, Q_R}{(1 - \rho r e)_R + Q_R} \right]$$

$$- \frac{I_o \, b_{pda}(k - \bar{r}_t e)(\rho k)^L \, k^{t-1} \, m_i Q_R}{(1 - \rho r_j e)[(1 - \rho r e)_R + Q_R]^2} \; ,$$

which, when coupled with a similar expression for brand j, gives the profit ratio

$$\pi_{R,eq} = m_R \left[\frac{[b_{pd}(PD)_i \, [(1 - \rho re)_R + Q_R] + b_{pda}\, Q_R]/(k - r_i e)}{[b_{pd}(PD)_j [(1 - \rho re)_R + Q_R] + b_{pda}(1 - \rho re)_R]/(k - r_j e)} \right.$$

$$\left. - \frac{b_{pda}\, Q_R(\rho k)^L/(1 - \rho r_j e)[(1 - \rho r e)_R + Q_R]^k}{b_{pda}\, Q_R(\rho k)^L(1 - \rho r e)^2_R/(1 - \rho r_i e)[(1 - \rho re)_R + Q_R]^k} \right] .$$

<u>INFERENCES FROM KUEHN'S OPTIMAL ADVERTISING DECISION RULE</u>

(1) At competitive equilibrium, the ratio of advertising expenditures between brands within a market segment (region or class of customers) is (a) directly proportional to the ratio of

their profit margins (m_R) and (b) inversely proportional to $(1 - \rho re)_R$, a ratio of consumer lack of loyalty to the brands modified by the effect of the discount factor ρ and the survival rate e.

(2) At competitive equilibrium, the ratio of advertising expenditures between brands is independent of the attraction of price, product characteristics, and retail availability (PD) of the brands upon potential brand shifters. Similarly, the ratio of advertising expenditures is independent of advertising effectiveness (E). These generalisations do not hold if PD or E for either brand is equal to 0. They must also be modified when the market is not at equilibrium.

(3) The level of ρk determines the effect of :

(a) Industry-wide levels of brand loyalty (r),

(b) rate of survival of customers from period to period (e),

(c) The time lag between advertising and its influence on sales (L),

upon the optimal level of advertising for the industry and any given firm. If $\rho k = 1$, these three variables do not influence the optimal level of advertising. If ρk is greater than 1, the optimal level of advertising is raised for all brands in the market when any one of the three variables is increased. If ρk is less than 1, the optimal level of advertising for the industry is decreased by an increase in the variables r, e, and L.

(4) Total industry advertising is directly proportional to the level of industry profit margins apart from advertising expenditures and to total industry sales volume when the market is at equilibrium. This generalisation applies to segments of the market as well as to the total market.

(5) For any given level of competitive profit margin (m) and brand loyalties (r), total industry advertising expenditures at equilibrium will be at a maximum when the ratio of advertising efficiencies (E_R) and the appeals of price, product characteristics, and retail distribution $(PD)_R$ are such that $E_R(PD)_R = (1 - \rho r e)_R / m_R$. Thus, given equal profit margins and brand loyalties, industry advertising expenditures will be at a maximum when competitors are equally matched in terms of advertising efficiency and the appeals of price - product - availability [i.e. $E_R(PD)_R = 1$]. The greater the difference in strength of competing brands, the lower the total industry advertising expenditures.

(6) An increase in the discount rate ρ (equivalent to a reduction in interest rate and risk) increases optimal advertising budgets since the investment in advertising can, in effect, be carried at lower cost. Within the model, part of the effect of advertising is treated as an investment since the customers attracted will tend to remain with the brand and produce sales and gross margin for an indefinite period of time estimated by the brand loyalty factor r.

(7) An increase in the net growth of an industry(k) increases optimal advertising expenditures but at less than a proportionate rate. Its effect depends upon the brand loyalty r, survival rate e, and the lag in the effect of advertising L.

(8) For competitors of total equal merchandising strength (i.e. $m_R\ E_R(PD)_R/(1 - \rho r e)_R = 1$), any deviation of competitive advertising expenditures from the equilibrium level requires the brand to reduce its advertising budget if it is to maximise profits given the competitive expenditures (More generally, including the case of two competitors of the equal merchandising strength) a brand's optimal budget will be at a maximum when the competitor budgets the fraction x of its optimal advertising expenditure such that $m_R\ E_R(PD)_R/(1 - \rho re)_R = 2x^{1/2} - 1$). The amount of reduction which is optimal is, however, much less than the amount of excess expenditures or underspending by competition. Consequently, a firm budgeting at equilibrium levels would be tolerably near optimal profitability over a wide range of competitive budgets if its cost structure, product appeals, retail shelf position, advertising effectiveness and price matches that of the competitor. The firm initially overbudgeting or underbudgeting advertising expenditures receives lower profits than it would have received at equilibrium. The firm responding so as to maximise its profits obtains greater-than-equilibrium profits if the competitor is under spending but suffers a decrease in profits if competition is overspending.

(9) Under equilibrium conditions, total industry advertising expenditures will increase as the number of brands in the market

increases. The effect of the number of competing brands upon the level of industry advertising expenditure can be represented by $(n-1)/nC_{eq,max}$, where $C_{eq,max}$ refers to total industry advertising expenditures at equilibrium given an infinite number of firms. Thus, two equally matched brands will spend only a little more than one-half as much as advertising as would a large number of firms serving the same market.

(10) Primary Advertising (advertising for the product class rather than for an individual brand of the product) is desirable for the brand holding advantages in perceived product characteristics, distribution, and costs of production (which can produce price and gross margin advantages). When brands are less well differentiated, it may be to each brand's advantage to join in co-operative advertising although it would be even more profitable to have competition foot the entire bill.

(11) If brand loyalty (r) varies cyclically over time, advertising expenditures should be increased to be at a peak when loyalty is at a minimum (assuming $L = 0$) and, conversely, should be at a minimum when loyalty is at a maximum.

(12) The equilibrium ratio of sales between brands in that portion of the market influenced by all merchandising variables including advertising (i.e., $b_{pda}\ I_t$), is directly proportional to the ratios of the brands profit margins m_R, advertising, effectiveness E_R, and the appeal of the brand's prices, product characteristics,

and retail availability $(PD)_R$, and inversely proportional to each of two slightly modified measures of brand disloyalty, $(k - re)_R$ and $(1 - \rho re)_R$. The market segments influenced only by price, product characteristics, and availability, but not by advertising (i.e., $b_{pd} I_t$), produce a ratio of sales between brands that is directly proportional to $(PD)_R$ and inversely proportional to $(k - re)_R$.

A NUMERICAL ILLUSTRATION OF THE KUEHN MODEL[34]

Suppose $I_o = 1{,}000$, $e = .9$, $g = .2$, $b_{pd} = .25$, $b_{pda} = .75$, $C_{i,o} = 1270$, $C_{j,o} = 1200$, $E_{i,o} = 1.4$, $E_{j,o} = 1.0$, $k - \bar{r}_1 e = .423$, $r_i = .8$, $S_{i,o} = 525$, $(PD)_i = .4$, $A_{i,o} = .597$, $r_j = .7$, $S_{j,o} = 475$, $(PD)_j = .6$, $A_{j,o} = .403$, $(PDA)_i = .239$, $(PDA)_j = .242$, $L = 1$, $T = 0$, $t = 1$, m_i \$ 5 per unit.

"Then, an advertising expenditure of Rs.1,270 will contribute \$ 785 sales Revenue at present, and \$ 2,015 sales Revenue in future periods, i.e., (in the future)"[35].

Consider two firms i and j, with two brands i and j, or two pairs of brands i and j.

Let m_j = \$ 6 per unit and $\rho = 0.94$.

The following table is obtained.

TABLE 6

Profits under various Advertising Strategies

Situation	Period 0 Advertising		Period 1 Unit sales		Period 1 unadjusted profits		Discounted value of period 0 advertising in future		Discounted Advertising Profit	
	i	j	i	j	i	j	i	j	i	j
i and j at equilibrium	1270	1200	577	523	1615	1938	2410	2352	1064	1080
j overspends i optimizes	1156	2000	531	569	1499	1414	1720	3024	495	904
j overspends i overspend	1500	2000	551	549	1255	1294	2025	2718	435	598
j overspends i underspends	1000	2000	519	581	1595	1486	1530	3204	470	1084
j underspends i optimizes	1229	800	607	493	1806	2158	2896	1896	1592	1048
j underspends i overspends	1500	800	620	480	1600	2080	3155	1722	1565	874
j underspends i underspends	800	800	572	528	2060	2368	2350	2424	1502	1576

THE VIDALE - WOLFE MODEL

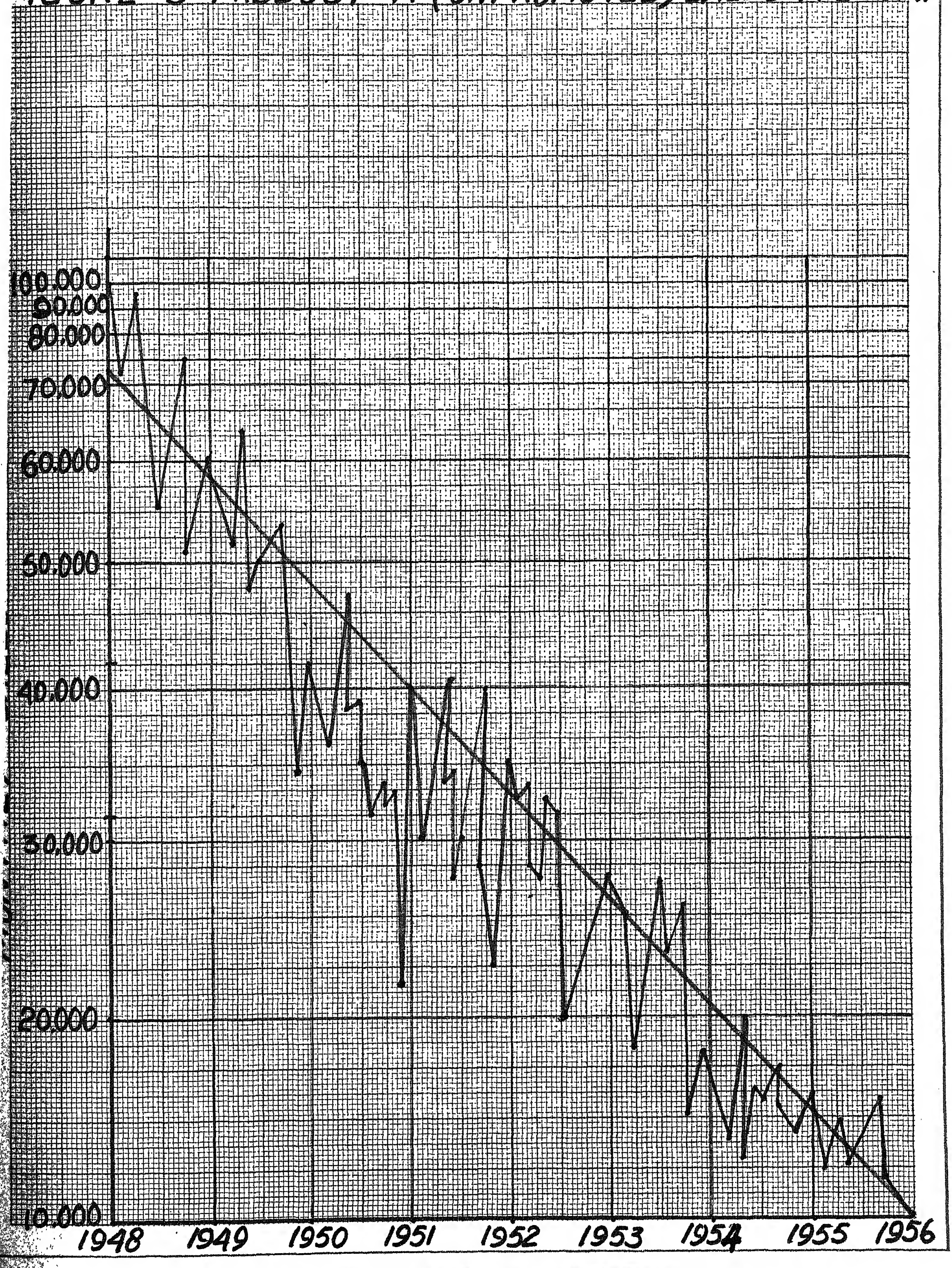
This model is the result of studies for seven major industrial concerns on the sales response to advertising. Vidale and Wolf has borrowed some of the operations Research techniques as applied to Advertising[36,37,38,39,40,41].

The Operations Research Group at Arthur D. Little, Inc., helped Vidale and Wolfe in gathering data from seven major industrial concerns. A large number of controlled experiments were performed, in which the intensity and type of promotion were varied. With the cooperation of sales and advertising departments and their advertising agencies, large-scale tests were conducted over considerable portions of the V.S. market. Vidale and Wolfe, in analysing advertising campaigns, described the interaction of sales and advertising in terms of three parameters : 1. The sales Decay Constant, 2. The Saturation level, and 3. The Response Constant[42].

THE SALES DECAY CONSTANT

In the absence of promotion, sales tend to decrease because of product obsolescence, competing advertising, etc. Under relatively constant market conditions, Vidale and Wolfe assumed, the rate of decrease is constant. Fig. 3 presents the eight-year sales history of product A, plotted on a semi-logarithmic scale. Product A exhibited a small seasonality in sales; however, over the years the sales decreased exponentially. Figure 4 represents the history of a

FIGURE-3-PRODUCT "A" (UNPROMOTED) SALES HISTORY



• ANNUAL AVERAGE OF MONTHLY SALES

FIGURE - 4 - UNPROMOTED PRODUCT "B" SALES HISTORY

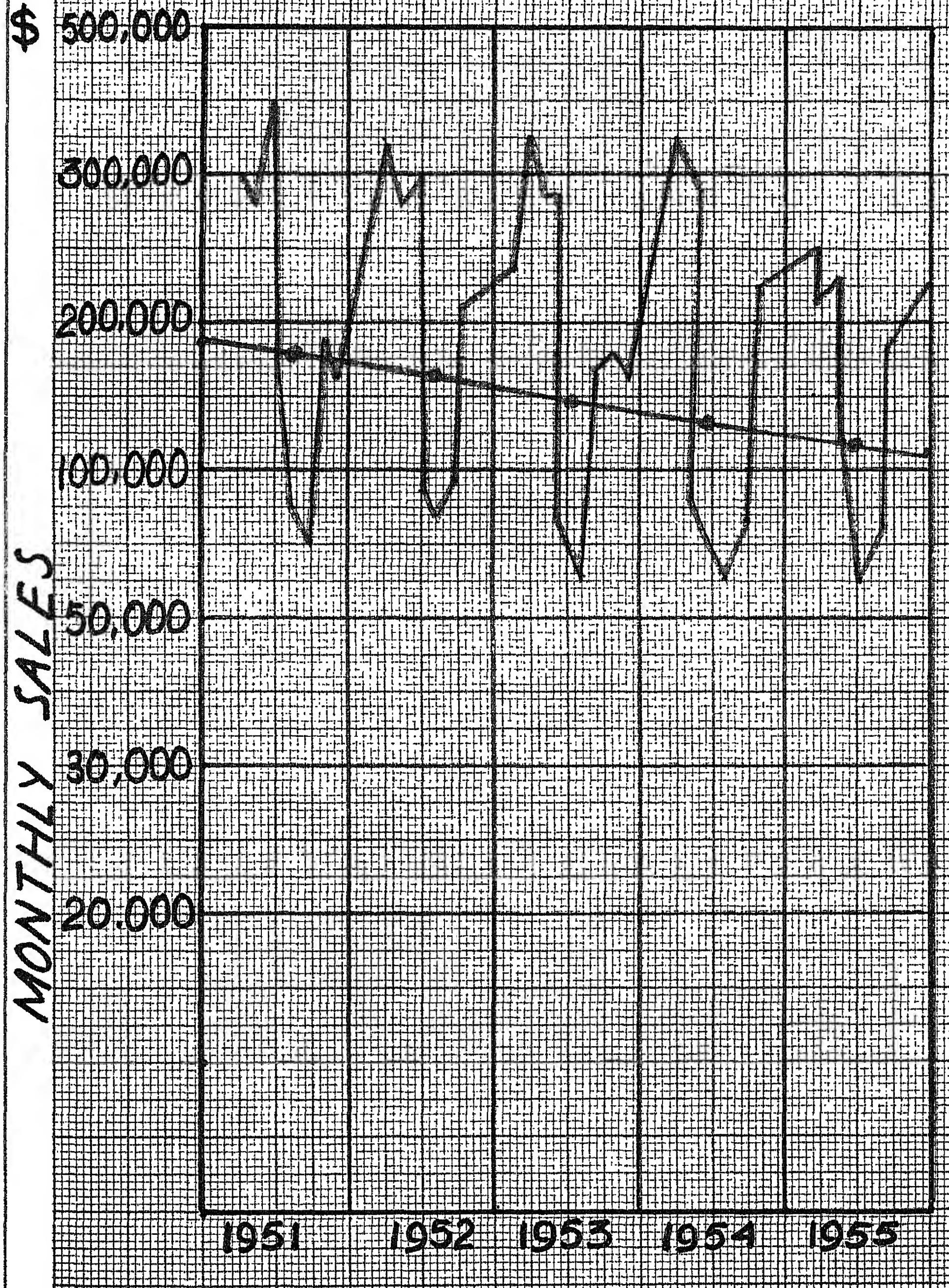
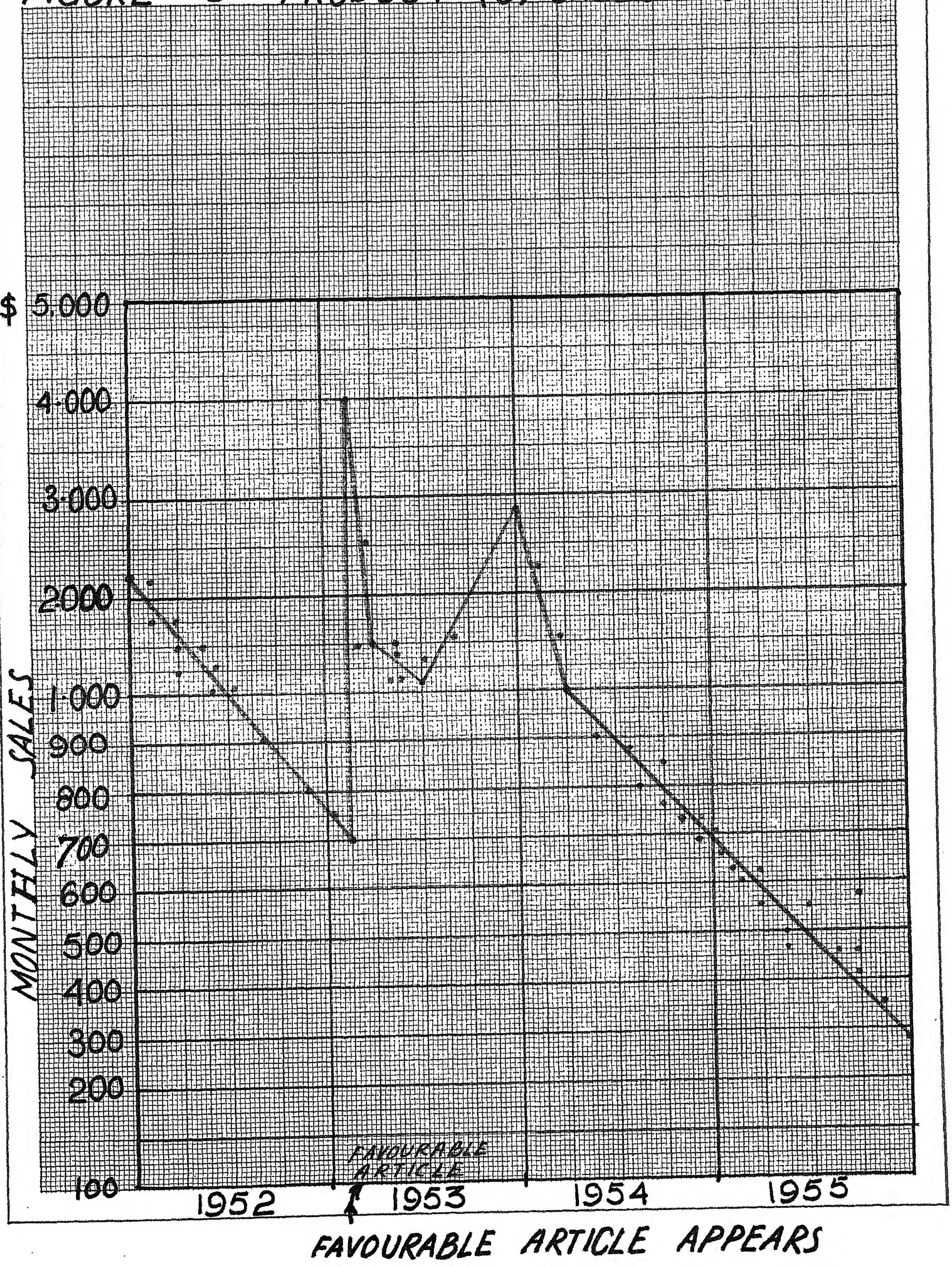


FIGURE - 5- PRODUCT (C) SALES HISTORY



very seasonal product, B. Here again, the monthly sales, averaged over a full year, "decay" at a constant rate.

Figure 3 - Product A (unpromoted) - Sales History

Figure 4 - Unpromoted Product B - sales history

This behavior, observed ina great number of unpromoted products, lead Vidale and Wolfe to postulate a sales Decay Constant λ; i.e. the sales rate at time t of an unpromoted product is given by $S(t) = S(o) \exp(-\lambda t)$. The sales decay rate ranges from large values for products that become quickly obsolescent or products in a highly competitive market to almost zero for noncompetitive, well established products.

Figure 5 illustrates the sales history of Product C. Sales of this product were decaying at a constant rate ($\lambda = 0.9$ per year) upto the beginning of 1953, when an article favourable to the product appeared in a popular magazine of wide circulation. Sales increased by a factor of five within a month. These level of sales, however, was not maintained, but began to decrease much more quickly ($\lambda = 4.7$ per year) than the original rate until it reached a new level, double that before the promotion. At this point, the sales decay constant returned to the original value of 0.9. Eight months later, the

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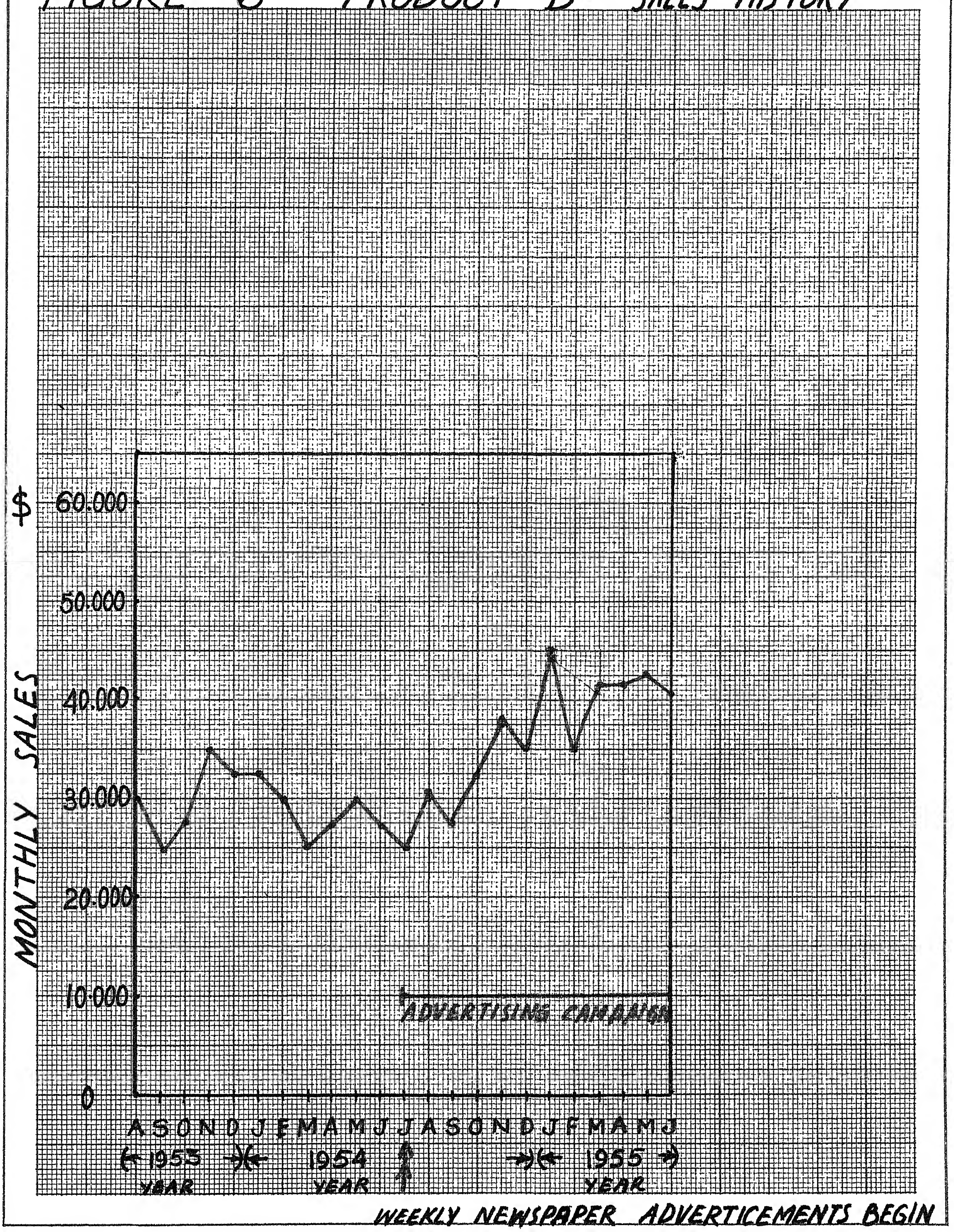
Figure 5 - Product C - Sales History

FIGURE —6— PRODUCT D- SALES HISTORY



product was mentioned favourably in another popular magazine, and the same phenomenon occured. Clearly, we are dealing here with two classes of customers, those who were induced to purchase after reading the magazine articles, but who soon lost interest in the product; and the "normal" customers, who behaved much like the original customer population.

THE SATURATION LEVEL

The concept of saturation level is illustrated by the sales history of product D (figure 6). This product was promoted continuously for one year by weekly newspaper advertisements begining in July 1954. In the first six month's, sales rose 30 percent and then leveled off, although the advertising campaign was continued for another six months. This additional advertising may have helped to maintain sales at the new level, but this effect cannot have been large, because the decay rate both before and after the advertising campaign was small. The campaign could have been considerably shorter and equally effective, and beyond a certain point, it lost its value.

Figure 6 - Product D - Sales History

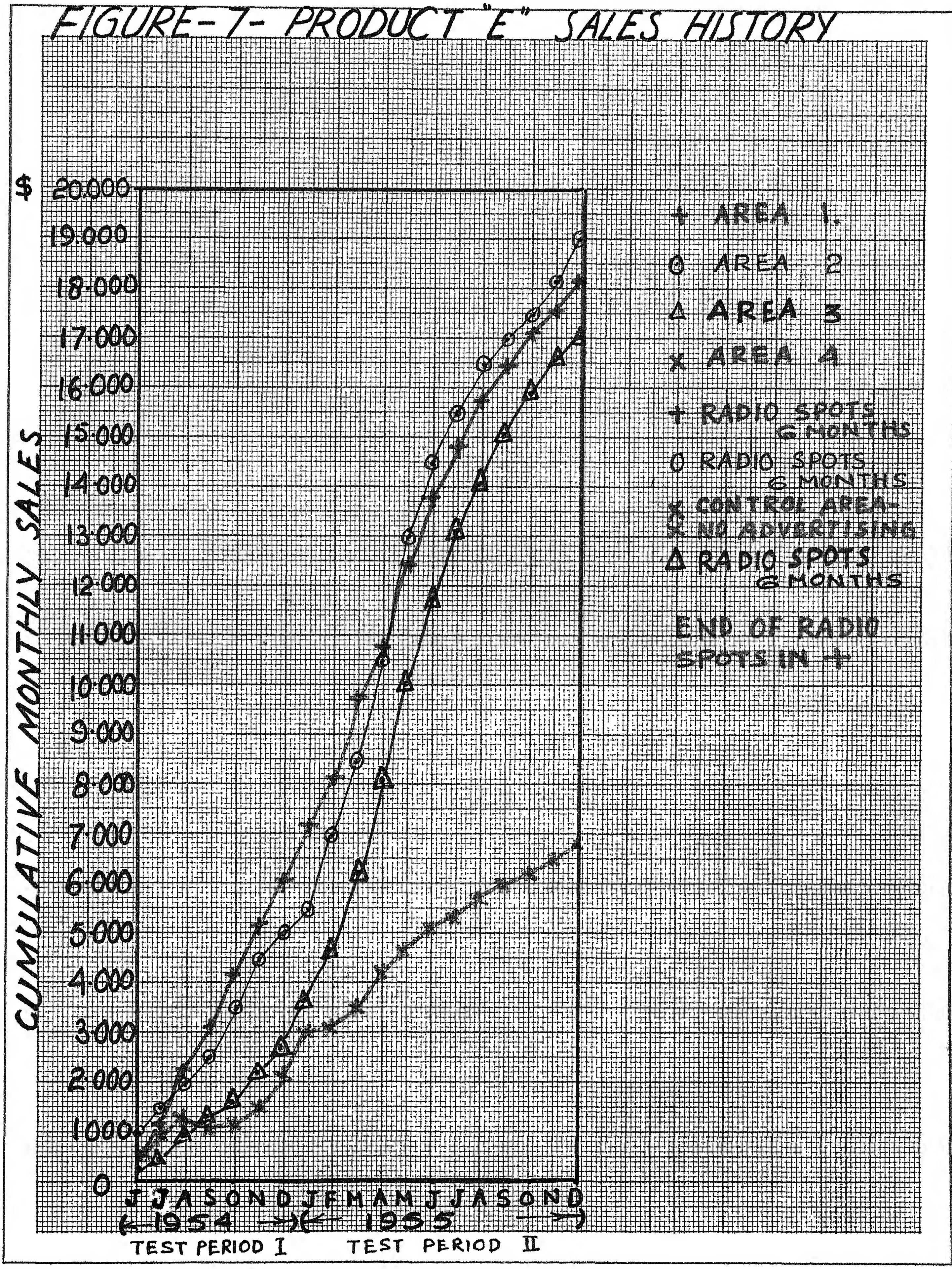


FIGURE-7- PRODUCT "E" SALES HISTORY
CUMULATIVE MONTHLY SALES
$ 20.000
19.000
18.000
17.000
16.000
15.000
14.000
13.000
12.000
11.000
10.000
9.000
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1000
0
J J A S O N D J F M A M J J A S O N D
1954
1955
TEST PERIOD I
TEST PERIOD II
+ AREA 1.
⊙ AREA 2
△ AREA 3
× AREA 4
+ RADIO SPOTS 6 MONTHS
⊙ RADIO SPOTS 6 MONTHS
× CONTROL AREA-
× NO ADVERTISING
△ RADIO SPOTS 6 MONTHS
END OF RADIO SPOTS IN +

Figure 7 presents the sales history of product E. Because of the complexity of the sales responses, sales were plotted on a cumulative scale. The product was observed in four areas. Area 1 received a spot radio commercial campaign for six months. Beginning at the same time, area 2 received the campaign for twelve months. At the end of the campaign in area 1, area 3 received the campaign for six months. Area 4 was kept as control and received no promotion.

Figure 7 - Product E - Sales History

In areas 1 and 2, sales increased approximately 150 per cent, over those in area 4; the additional six month's promotion received by area 2 did not increase sales further. Area 3 experienced a similar sales increase after the promotions started. Therefore, even though the advertising campaign was postponed for six months, it lost none of its effectiveness.

Figures 6 and 7 lead Vidale and Wolfe to postulate a Saturation level M, or practical limit of sales that can be generated. This saturation level depends not only on the product being promoted, but also on the advertising medium used; it represents the fraction of the market that the particular campaign can capture; this saturation level can often be raised further by other advertising media.

THE RESPONSE CONSTANT

The Response Constant r is defined as the sales generated for monetary unit of advertising when $S = 0$. The number of new customers who are potential buyers decreases as sales approach saturation. When advertising is directed indiscriminately to both customers and non-customers, the effectiveness of each advertising rupee in obtaining new customers also decrease as sales increase. In general, the sales generated per advertising rupee, when sales are at a level S, is given by $\frac{r(M-S)}{M}$, where M is the saturation level.

MATHEMATICAL MODEL OF VIDALE AND WOLFE

The Vidale-Wolfe mathematical model is represented by

$$\frac{dS}{dt} = rA(t)\,\frac{(M-S)}{M} - \lambda S \quad ,$$

where
- λ = Sales Decay constant,
- M = The Saturation level
- r = Response Constant
- S = Rate of sales at time t
- $A(t)$ = Rate of Advertising Expenditure
- $\frac{dS}{dt}$ = Increase in the rate of sales
- A = Intensity of Advertising expenditure.

Thus, the increase in the rate of sales is proportional to the

intensity of the advertising effort reaching the fraction of potential customers $\frac{M-S}{M}$, less the customers that are lost λS.

INFERENCES FROM THE VIDALE-WOLFE MODEL

(1) Steady-state Solution. The advertising effort required to maintain sales at a steady predetermined level can be determined by putting $\frac{dS}{dt} = 0$.

Then,
$$A = \left(\frac{\lambda}{r}\right)\frac{SM}{M-S}$$

Thus, the closer sales are to the saturation level M, and the larger the ratio $\frac{\lambda}{r}$, the more expensive it is to maintain the required sales rate.

(2) For a constant rate, A, of advertising expenditure, maintained for time T, the rate of sales is obtained by integration of the equation $\frac{dS}{dt} = rA(t)\frac{(M-S)}{M} - \lambda S$.

For $t < T$

$$S(t) = \left[M/(1 + \lambda M/rA)\right]\left\{1 - e^{-(rA/M+\lambda)t}\right\} + S_o\, e^{-(rA/M+\lambda)t},$$

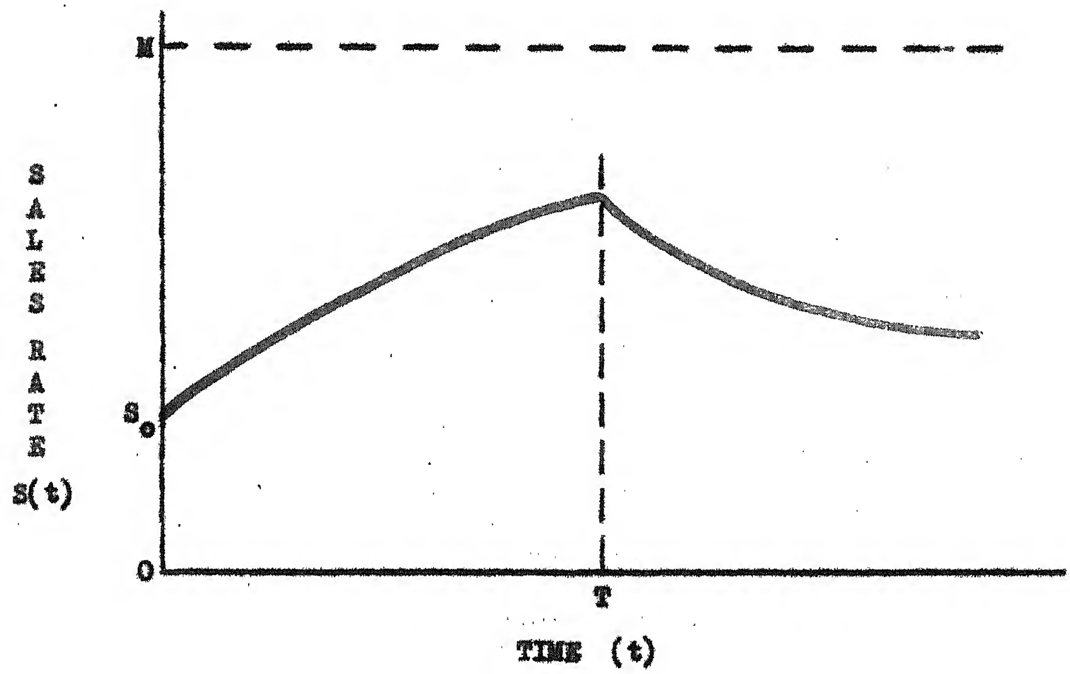
where S_o is the rate of sales at $t = 0$, the start of the advertising campaign. After advertising has stopped,

For $t > T$,

$$S(t) = S(T)\, e^{-\lambda(t-T)} .$$

The sales response to an advertising campaign of constant intensity and of duration T is shown in figure 8.

Figure 8 - Sales Response to an advertising campaign of duration T.



The rate of sales increase is most rapid at $t = 0$; as saturation, M, is approached, this rate is reduced. This means that the first advertising rupee expended is most effective, the second rupee is next most effective, and so on. A second implication is that for equal expenditures, a protracted advertising campaign is more profitable than a short intense one.

(3) Advertising pulse

$$\text{Let} \qquad A(t) = \begin{cases} \frac{a}{T} & \text{for } 0 < t < T \\ 0 & t > T \end{cases} .$$

This means total expenditure, a_j, spent at a constant rate for time T. The rate increases as T decreases. We get "single-pulse campaign of negligible duration" which is "intense" by letting $T \to 0$. Solution of the eqn. $\frac{dS}{dt} = rA(t)\,\frac{(M-S)}{M} - \lambda S$ for this $A(t)$ is found by replacing the constant A in the expressions for $S(t)$ by $\frac{a}{T}$. The height of the graph in figure 8, at its heighest point, i.e., at $t = T$, is

$$S(T) = \frac{M}{1 + \frac{\lambda M T}{ra}} \left\{ 1 - e^{-\left(\frac{ra}{TM} + \lambda\right)T} \right\} + S_o\, e^{-\left(\frac{ra}{TM} + \lambda\right)T} .$$

Now let $T \to o$,

$$\lim_{T \to o} S(T) = M \left\{ 1 - e^{-\frac{ra}{M}} \right\} + S_o\, e^{-\frac{ra}{M}}$$

$$= M - (M - S_o)\, e^{-\frac{ra}{M}} .$$

We thus find the new level of sales rate to which sales jump instantaneously due to the single-pulse injection of advertising at $t = 0$. From this point on, this new level decays exponentially. Multiplying $\lim_{T \to 0} S(T)$ by $e^{-\lambda t}$, we get

$$S(t) = M\, e^{-\lambda t} - (M - S_o)\, e^{-(ra/M+\lambda)t} ;$$

This is a satisfactory impression for a short, very intense, single-pulse campaign. The immediate sales increase resulting from the promotion is

$$S(o) - S_o = (M - S_o)\left(1 - e^{-ra/M}\right) .$$

The total additional sales generated by this campaign are

$$\int_o^{\infty} \left[S(o) - S_o\right] e^{-\lambda t}\, dt = \frac{M - S_o}{\lambda}\left(1 - e^{-ra/M}\right) ,$$

which reduces to $\left(\frac{ra}{\lambda}\right)(M - S_o)/M$ for sales well below saturation. The total extra sales generated by the advertising campaign are therefore the immediate sales increase, multiplied by the mean life of the product, λ^{-1}. Also, given a choice of several products, the advertising campaign will generate the most sales for the product with the largest value of $\left(\frac{r}{\lambda}\right)(M - S_o)/M$.

To compare the sales increase from a short, intense campaign with that from a longer, less intense one we may compare the sales increase which results when we spend at the constant rate of 2A per period for T periods, (short, intense campaign) with that which results when we spend at the constant rate of A per period for 2T periods (longer, less intense campaign). The sales increase which results when we spend at the rate A for T periods is

$$\int_0^T S(t)\,dt = \int_0^T \frac{rAM}{rA + \lambda M}\,dt + \int_0^T S_o\, e^{-\left(\frac{rA}{M} + \lambda\right)t}\,dt$$

$$- \int_0^T \frac{rAM}{rA + \lambda M}\, e^{-\left(\frac{rA}{M} + \lambda\right)t}\,dt$$

$$= \frac{rAMT}{rA + \lambda M} - \frac{M S_o\, e^{-\left(\frac{rA}{M} + \lambda\right)T}}{rA + \lambda M} + \frac{MS_o}{rA + \lambda M}$$

$$+ \frac{rAM^2\, e^{-\left(\frac{rA}{M} + \lambda\right)T}}{(rA + \lambda M)^2} - \frac{rAM^2}{(rA + \lambda M)^2}$$

Dropping the λM, since (it is the same at 2A for T periods and A for 2T periods), it is uninfluential in the comparison, and simplifying, we have

$$\frac{rAMT + M S_o\left(1 - e^{-\frac{rAT}{M}}\right) + M^2\left(e^{-\frac{rAT}{M}} - 1\right)}{rA} .$$

Comparing :

$$\text{* (short, intense)} \quad \frac{MT + MS_o\left(1 - e^{-\frac{r2AT}{M}}\right) + M^2\left(e^{-\frac{r2AT}{M}} - 1\right)}{r\,2A}$$

$$\text{and ** (longer)} \quad \frac{M2T + MS_o\left(1 - e^{-\frac{rA2T}{M}}\right) + M^2\left(e^{-\frac{rA2T}{M}} - 1\right)}{rA} ,$$

we find that the longer, less intense campaign does indeed produce a substantially greater sales increase per rupee spent for advertising.

(4) <u>Allocation of Advertising Budget</u> :

Those products should be advertised that will result in a return on capital invested equal to or greater than the returns from other possible investments, such as new equipment and research.

Considering a case of a family of products that might be advertised by short, intense campaigns, if

a_k = total cost of the proposed advertising campaign for product k.

$R_k(t)$ = additional sales resulting from the advertising campaign.

$C_k(t)$ = rate of additional expenditures resulting from the advertising campaign. These include (a) the cost of the advertising campaign itself, (b) the cost of manufacturing and distributing the additional items sold.

I_k = return on capital invested in advertising products k,

then, the sum total of expenditures incurred by the promotion of product k discounted at the rate I_k from the start of the advertising campaign (t = 0) is

$$\int_0^\infty C_k(t)\, e^{-I_k t}\, dt \ .$$

The additional sales resulting from the advertising campaign, also discounted at the rate I_k, are

$$\int_0^\infty R_k(t)\, e^{-I_k t}\, dt .$$

In order to determine the rate of return on capital invested in the promotion of product k, the expenditures and sales increases are equated :

$$\int_0^\infty C_k(t)\, e^{-I_k t}\, dt = \int R_k(t)\, e^{-I_k t}\, dt .$$

Under the assumption that production and distribution costs are proportional to sales, we have

$$C_k(t) = f_k\, R_k(t) + a_k ,$$

where f_k is the ratio of production and distribution costs to selling price. Assuming that the rate of sales of the unpromoted product decays exponentially at the rate λ_k, we have

$$R_k(t) = R_{ok}\, e^{-\lambda_k t} ,$$

where R_{ok} is the instantaneous sales increase resulting from the campaign. Substituting the expressions for $C_k(t)$ and $R_k(t)$ in the expenditures - sales increase equality, we have

$$a_k + \int_0^\infty f_k\, R_{ok}\, e^{-\lambda_k t}\, e^{-I_k t}\, dt = \int_0^\infty R_{ok}\, e^{-\lambda_k t}\, e^{-I_k t}\, dt .$$

Integrating and solving for I_k,

$$I_k = \left(\frac{R_{ok}}{ak} \right) (1 - f_k) - \lambda_k \ .$$

The rate of return I_k is a function of the intensity of the advertising campaign. Once the values of I_k are known, one can select the products that may be advertised profitably. Thus, the amount of advertising appropriate to each product, and consequently the total advertising appropriation, can be determined once the I_k are known.

SHORTCOMINGS OF THE VIDALE- WOLFE MODEL

The assumption that the rate of response to advertising is constant per rupee spent is not realistic. Such an assumption implies the absence of competitive reaction to increased sales resulting from advertising.

Vidale and Wolfe have assumed that advertising merely obtains new customers. Increased customer usage of the product is neglected. The model also abstracts out possible variations in the effectiveness of advertising expenditure. In other words, two campaigns using different themes and appeals are assumed to be equally effective if the total cost of each is equal. Media effectiveness[43] and message effectiveness[44] are ignored.

RETURN ON ADVERTISING INVESTMENT IN THE KUEHN MODEL AND IN THE VIDALE-WOLFE MODEL

In the Vidale-Wolfe model, the return on the advertising investment is given by : $I = \frac{R_o}{a}(1 - f) - \lambda$, where R_o is the instantaneous sales response measured in monetary units, a is the advertising expenditure, f is the ratio of production and distribution costs to selling price, and λ is the decay constant. In the Kuehn model, $R_o(1 - f)$ corresponds roughly to $m_i S(A)_t$, a corresponds to $C_{i,T}$ and λ corresponds to $1 - r_i e$. In the Kuehn model, if the return on investment is defined as the rate of discount which equates expected return with cost,

$$m_i S(A)_t \sum_{t=o}^{\infty} (r_i e \phi)^t = \frac{C_{i,T}}{\phi^L} ,$$

or,

$$m_i S(A)_t \sum_{t=o}^{\infty} \left(\frac{1 - \lambda}{1 + I}\right)^t = C_{i,T}(1 + I)^L .$$

Solving for I we find :

$$I_i = \frac{m_i S(A)_t (1 + I)}{C_{i,T}(1 + I)^L} - \lambda$$

Thus, if L = 1, the Kuehn model gives the same return on advertising investment as that given by the Vidale-Wolfe model.

CUMULATED IMPRESSION MODEL

The psychology of advertising was investigated by Walter Drill Scott (1903) and Lucas and Britt (1963). Lucas and Britt summarised their findings of experiments on memory. "Repetition of advertising has advantages in memory other than the increased chance of retention. Repetition reinforces and strengthens the impressions made on the audience. Each time an idea is repeated, the impression becomes stronger. Each time an impression is reestablished, it tends to last longer ....

"Each time an impression is repeated, it becomes stronger in two ways. First, it builds to a higher level than the original by combining the influence of the new with the residue of the old. Secondly, there is a definite tendency for impression established through space repetitions to fade more slowly in each successive stage".

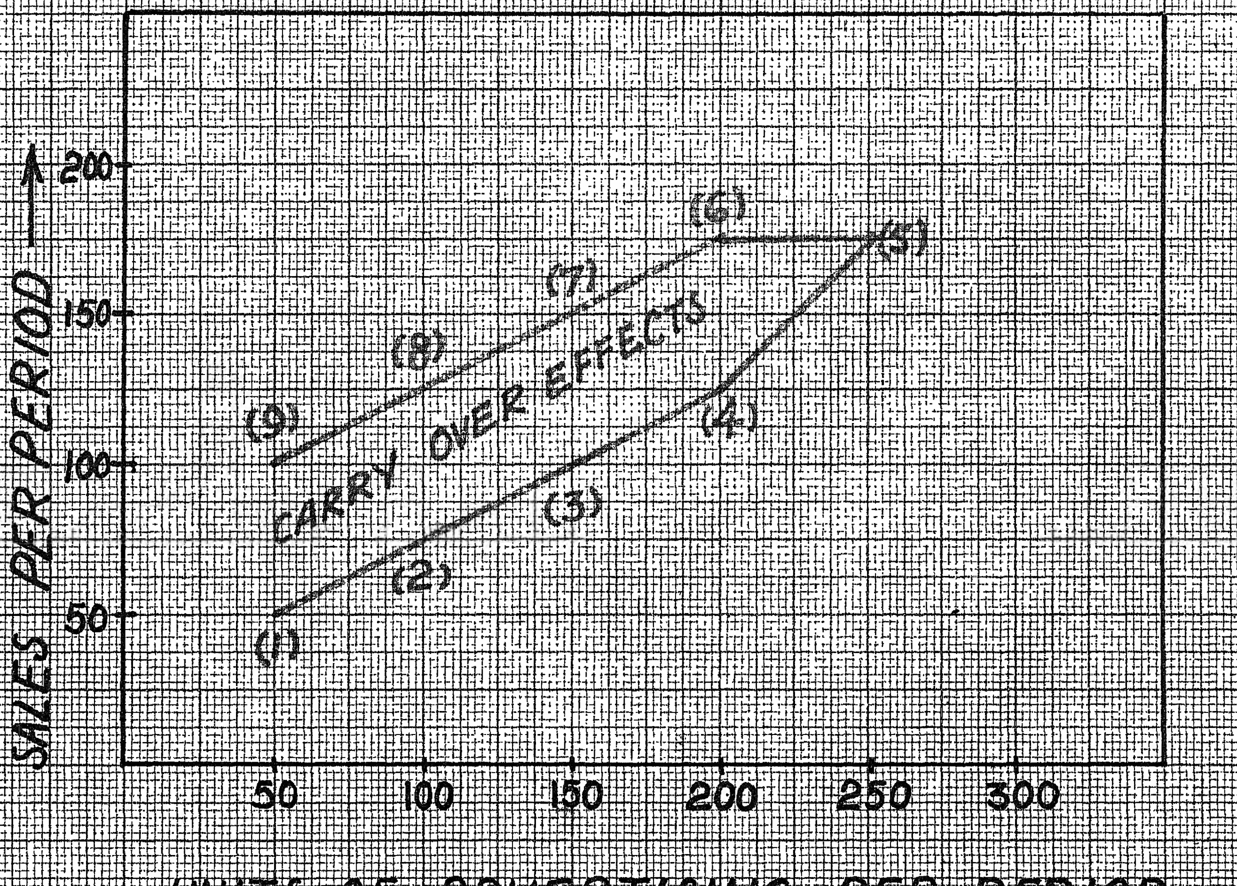
Thus, the level of awareness of brand and of product attributes may be built up over time with advertising. If the level of Sales is a function of brand awareness, present-period sales become a function of both present - and past - period advertising.

Donald S. Tull, in 1965, constructed a simple mathematical model[45].

$$Y_n = F\left(A_n, A_{n-1}, \ldots\ldots\ldots, A_{n-j}\right)$$

where Y_n = given period sales, and

FIGURE - 9 - "CUMULATED IMPRESSION" MODEL RESULTS (After Tull



A_n = given period advertising.

If a linear relationship is assumed,

$$Y_n = a + b\left(A_n, A_{n-1}, \ldots\ldots\ldots, A_{n-j}\right)$$

where a = level of sales without advertising,

b = change in given period sales per increment of cumulated advertising.

Further, only given-period and immediately-preceding period advertising are assumed to affect the given-period sales. It is further assumed that the effect of the previous-period advertising on given period sales will be reduced by a constant decay factor of 1/C. The model simplifies to

$$Y_n = a + b\left(A_n + \frac{A_{n-1}}{c}\right).$$

Figure 9 and Table 7 show the operation of the model.

Figure 9 - "Cumulated Impression" model results (after Tull)

THE ZENTLER - RYDE MODEL

A.P. Zentler and Dorothy Ryde considered a private International organisation. They first developed the general form of a curve representing response to advertising in favour of one commodity only, say X, in competition with a substitute X'. They then considered the

TABLE 7

Operation of Cumulated Impression Model (after Tull)

Period	Advertising A_n	a	$b\left(A_n + \frac{A_{n-1}}{c}\right)$	Sales
1	50	50	15	65
2	100	50	38	88
3	150	50	60	110
4	200	50	82	132
5	250	50	105	165
6	200	50	98	148
7	150	50	75	125
8	100	50	52	102
9	50	50	30	80

problem - suppose the organisation has branches in n different countries - How should the organisation allocate its advertising expenditure among them so as to get the maximum overall return?

In developing the general form of a curve representing response to advertising in favour of one commodity only, response is defined as the increase in the consumption of X above its natural level[46], by which is meant the consumption of X that would exist if there was no promotion for either X or X'. It is assumed that inter-country differences in response depend on the following parameters: population, relative publicity and other costs, per capita consumption (X + X'), and a psychological factor θ (which measures the "X-

mindedness" of the population); these are introduced into the general equation of the response curve. Allowance is also made for a time-lag in the effects of Advertising. The "interaction effect" of simultaneous advertising in favour of both X and X' is then considered and a formula is developed for the resultant response R in this case. All the parameters except θ are capable of direct measurement; θ can be measured indirectly from the known percentage of X in $(X + X')$, that results from known advertising expenditure.

The optimum allocation is defined as that which maximise ΣR summed over all countries. If past advertising in favour of X and both past and current advertising in favour of X' are treated as given constants, R is a function of the single variable x, representing current advertising expenditure for X. The maximisation of ΣR, with the condition Σx constant, has been dealt with in fairly great detail by Zentler and Ryde.

Zentler and Ryde have assumed that within one country, the advertising budget is spent in the most effective manner, and they have tackled the problem of finding the best way to allocate expenditures between countries.

The Zentler-Ryde analysis refers to a situation in which the established market for commodity X is being "invaded" by a new substitute. In such a case there are two possible results of promotional activity; the proportion in which the market for this type of commodity is shared between the two competing products could be changed, without affecting the total demand for the type of

commodity under consideration; or the total demand itself could be expanded. In real life both changes will usually take place concomitantly, but it is obviously a much more difficult task for advertising to effect a fundamental change in taste, say from drinking Coca-Cola regularly to reading books, than to bring about a shift from one product to a close substitute. On these grounds, Zentler and Ryde decided to neglect the secondary effect of publicity, i.e., the increase in the total demand for the type of commodity under consideration.

In these circumstances, Zentler and Ryde supposed the mere existence of the competing product will lead to a certain amount of substitution, even without advertising the extent of this spontaneous advertising has been taken as an inverse measure of the amount of psychological resistance to the new product that exists naturally. This will vary from country to country according to the tastes etc. of the inhabitants and, in any one country, will also depend on the relative prices, there, of the two competiting commodities.

If the effect of promotion on behalf of one commodity only is considered, and if it is assumed that, at this stage, there is no time lag in the effect of advertising, then any change reaches its full impact immediately. Since the unit time period has been taken tobe one year, any change in advertising is assumed to achieve its full effect in less than one year.

Figure 10 - The Zentler Ryde Response Curve for an individual

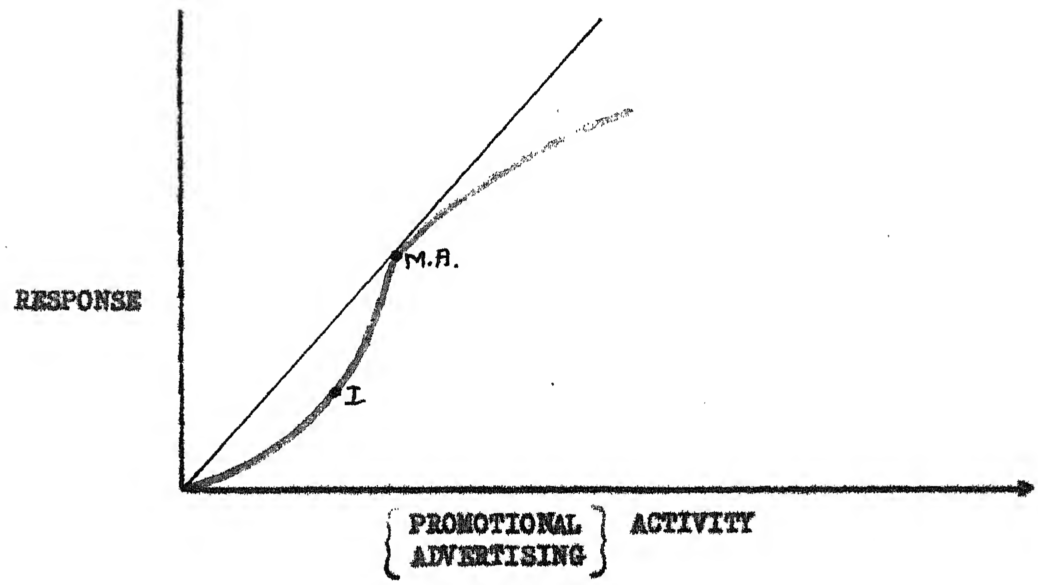


Figure 10 shows the individual's response to steadily increasing advertising activity. When advertising is first started, the response is small, but, once the required "softening up" process has been performed there is a range in which response rises rapidly as promotional activity is increased. Ultimately, as promotion is increased to much higher levels, the rate of increase in response tails off again and a point is reached at which further promotion produces very little additional effect. A curve of this type has two important characteristic points; the point of inflexion I, and the point M.A., at which the average response is a maximum. The later (the M.A. point) is, of course, also the point at which the average response is equal to the marginal response, geometrically it is the point of contact of the tangent from the origin.

ALGEBRAIC FORM FOR THE RESPONSE CURVE

The general algebraic expression for the Response Curve is

$$R = A\,\frac{c_2\xi^2 + c_3\xi^3 + \cdots\cdots + c_n\xi^n}{1 + d_1\xi + d_2\xi^2 + d_3\xi^3 + \cdots\cdots + d_n\xi^n} \quad ,$$

where R is response, ξ is expenditure on advertising and A is the limiting value that R approaches as ξ becomes infinitely large; n can have any value ≥ 2 but the coefficients must obey the conditions

$$d_1 \geq 0\ ,\quad c_n = d_n,\ c_s \leq d_s \quad \text{for all } 2 < s < n\ ,$$

and other more complicated conditions that are imposed by the fact that R must always increase with ξ.

All the curves of this family start off at grazing incidence to the ξ axis at the origin and rise, at first more rapidly and then more gradually, until they finally approach the limit A assymptotically.

If the behavior of the curve at two or three points is specified, and the constraints are restricted accordingly, the amount of variation left is quite small; though in theory we have an infinite number of possible curves these will differ only in a "second order" way and this kind of difference will not affect the allocation very much. By taking n large enough and adjusting the constants suitably we could, if we wished, control this second order behaviour very accurately. However, since the wanted- behaviour is

not known in great detail, all the conditions can be satisfied by the simple form

$$R = A \frac{c_2 \xi^2}{1 + d_1 \xi + d_2 \xi^2}$$

If we impose the condition that the M.A. point shall correspond to an expenditure β we find

$$c_2 = \frac{1}{\beta^2}$$

and the equation becomes

$$R = A \frac{(\xi/\beta)^2}{1 + d_1 \xi + (\xi/\beta)^2}$$

which may also be written

$$R = A \frac{(\xi/\beta)^2}{1 + d_1'(\xi/\beta) + (\xi/\beta)^2}$$

There is one more constant d_1', and this is fixed with reference to the steepness of the curve. Zentler and Ryde did not expect the average response to have a very sharp maximum at the M.A. point. The larger d_1' is made, the flatter will be the "corner" at the M.A. point and the lower (relatively to A) will be the actual response at this point.

Some members of the curve family represented by the last equation are sketched in Figure 11.

FIGURE - II - MEMBERS OF THE CURVE FAMILY REPRESENTED

$$R = \frac{A(\xi/\beta)^2}{1 + d_1'(\xi/\beta) + (\xi/\beta)^2}$$

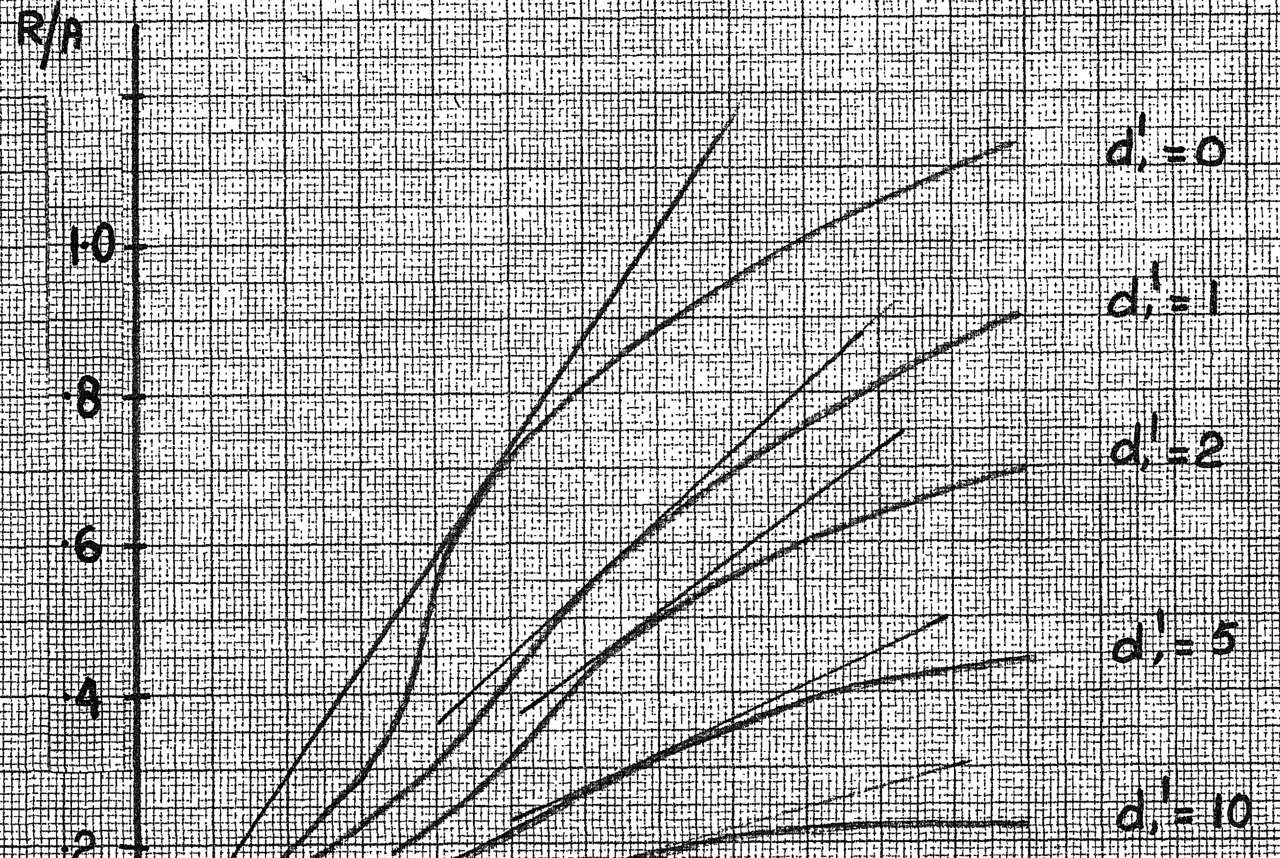


Figure 11 - Members of the curve family represented by

$$R = A \frac{(\ell/\beta)^2}{1 + d_1'(\ell/\beta) + (\ell/\beta)^2}$$

For larger values of d_1' the "corner" has almost vanished and the curves are so flat as almost to contradict the assumption of diminishing returns. Also, the actual proportion of A that can be reached, even by an expenditure many times that necessary to achieve the maximum average, is still very small.

The lower values of d_1' seem to give a more reasonable falling off in the rate of response. $d_1' = 0$ was rejected on the ground that it approaches A rather two abruptly and $d_1' = 1$ was chosen by Zentler and Ryde as the "best" value. Since the same basic form for response curves was taken in all countries, choosing a wrong d_1' would not be significant with $d_1' = 1$,

$$R = A \frac{(\ell/\beta)^2}{1 + (\ell/\beta) + (\ell/\beta)^2} .$$

Zentler and Ryde then went on to consider how the curves vary from country to country. They assumed that the following parameters, are responsible for variation between countries.

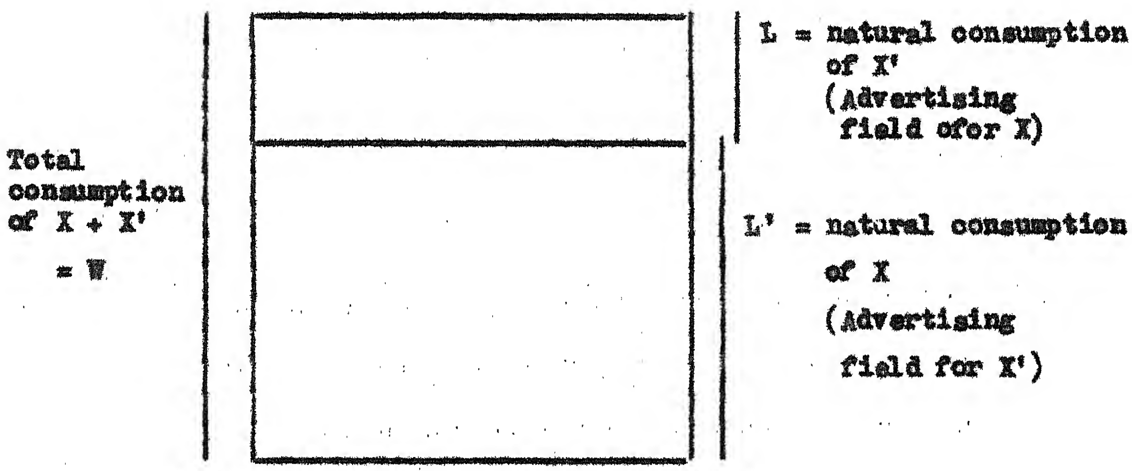
1. The total consumption per head of (X + X').
2. A psychological factor which we have called "X-mindedness",

measuring the extent to which people are favourably disposed towards X or resistant to the substitution of X by X'.

3. Cost of Advertising in terms of a common momentary denominator.

4. Population.

Figure 12



If "X-mindedness" is denoted by θ and "X'-mindedness" by θ' then, as already indicated, we take L'/W and L/W as the measures of θ and θ' so that

$$\theta = L'/W \qquad \theta' = L/W \qquad \theta + \theta' = 1 .$$

If "response" is defined as the increase in consumption of X above the natural or unpromoted level L', then it is clear that L represents a physical upper limit to R and should be identified with A of the equation. Then,

$$R = L \frac{(\xi/\beta)^2}{1 + (\xi/\beta) + (\xi/\beta)^2}$$

Zentler and Ryde expected that response would be a percentagewise phenomenon, i.e., that a small consumer and a large consumer would increase their consumption by the same percentage, rather than by the same absolute amount, in response to a given amount of advertising. This implied that, other things being equal, response would be proportional not only to W, but to X for X promotion and to X' for X' promotion : There will be some modification from country to country due to varying X-mindedness. The greater the X-mindedness of a countiy (i.e. the larger θ) the less X is likely to have been damaged (in that country) by the X' invasion and hence the less room X has to expand.

This additional condition is satisfied by the equation

$$R = L\frac{(\xi/\beta)^2}{1 + (\xi/\beta) + (\xi/\beta)^2} ,$$

since $L = W(1-\theta)$.

This, however, is not the only effect that X-mindedness will have on the equation; θ will also have an effect on β. Since β is the amount of real expenditure required to produce a specific result in each country it will be less the greater the X-mindedness of the country, because it should be easier to produce a given result in an X-minded country. Zentler and Ryde, therefore, assumed that β is inversely proportional to θ and wrote $\beta = \frac{b}{\theta}$, where b is a constant for all countries.

A factor P expressing advertising "power" per unit expenditure

in that country was constructed. If x represents expenditure on advertising, Px would be the corresponding real advertising.

Zentler and Ryde assumed that the response of N people is N times that of one person. (This is not necessarily true - for instance, the proportionate frequencies of those more susceptible and less susceptible to advertising may vary from country to country). This assumption leads to the view that per capita response curves will not be affected by differences in population.

Hence the curve representing per capita response to varying amounts of advertising expenditure in favour of one commodity only may be represented by the formula

$$R = L \frac{\left(\frac{\theta Px}{b} \right)^2}{1 + \left(\frac{\theta Px}{b} \right) + \left(\frac{\theta Px}{b} \right)^2} = L \phi \left(\frac{\theta Px}{b} \right)$$

where R = response per capita

x = advertising expenditure per capita

θ = "X-mindedness"

L = maximum physically possible response

b = a constant scale factor.

TIME LAGS

Supposing that instead of reaching its full effect immediately, a fixed annual expenditure results in a response that increases gradually towards a limiting value, which is denoted by

the response curve derived above, then

$$\phi\left(\frac{\theta Px}{b}\right) = \phi(\epsilon) \quad .$$

On the new basis $R_t = L\phi(\epsilon)\,\phi(t)$

where $\phi(t)$ is an increasing function of t, which approaches unity for large t. $\phi(t)$ may be generated thus: in each time interval response increases by a constant fraction, α of the remaining possible increase.

Then,

$$\Delta R_t = L\,\phi(\epsilon)\,\alpha\left[1 - \phi(t)\right]$$

$$= \alpha L\,\phi(\epsilon) - \alpha R_t$$

$$R_{t-1} - R_t = \alpha L\phi(\epsilon) - \alpha R_t$$

$$R_{t+1} = \alpha L\phi(\epsilon) + (1 - \alpha)\,R_t$$

By repeated substitution :

$$R_{t+1} = \alpha L\phi(\epsilon)\left\{1 + (1-\alpha) + (1-\alpha)^2 + \dots + (1-\alpha)^t\right\}$$

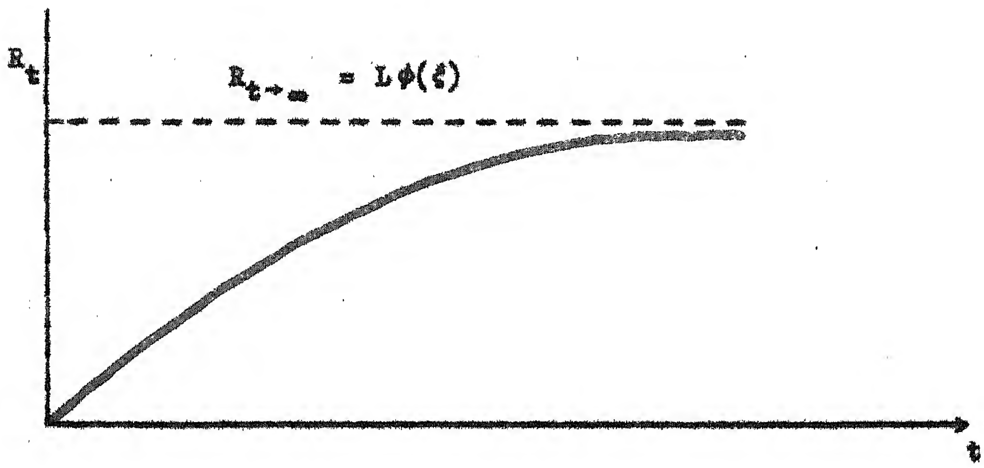
$$= L\phi(\epsilon)\left[1 - (1-\alpha)^{t+1}\right]$$

$$R_t = L\phi(\epsilon)\left[1 - (1-\alpha)^t\right]$$

which implies that

$$\phi(t) = 1 - (1 - \alpha)^t$$

Figure 13



VARYING ξ

Supposing that the generating process holds when there is a difference ξ in each time period, then

$$R_t = \alpha L \phi(\xi_t) + (1 - \alpha) R_{t-1}$$

which leads, by successive substitution, to

$$R_t = \alpha L \Big\{ \phi(\xi_t) + (1 - \alpha)\phi(\xi_{t-1}) + (1 - \alpha)^2 \phi(\xi_{t-2}) \cdots + (1 - \alpha)^{t-1} \phi(\xi_1) \Big\},$$

which reduces to

$$R_t = L\phi(\xi)\left[1 - (1 - \alpha)^t\right] \quad \text{if all } \xi \text{ are equal.}$$

In practice, since $\alpha < 1$ the series converges quite rapidly (specially if ξ increases with time) and it is only necessary to consider the first few terms.

The model has taken no account of the possibility that prolonged advertising may produce an irreversible change in taste, such as would be represented in our model by a change in θ. This does not, however, restrict the usefulness of the model very much, as we are not really concerned with very long term changes. Furthermore, since at any time, the current value of θ is observed, irreversible changes are automatically allowed for.

SIMULTANEOUS ADVERTISING IN FAVOUR OF BOTH COMMODITIES

On the assumption of simultaneous response, the response to advertising for commodity X is

$$R = L\phi(\xi) .$$

For X',

$$R' = L'\phi(\xi')$$

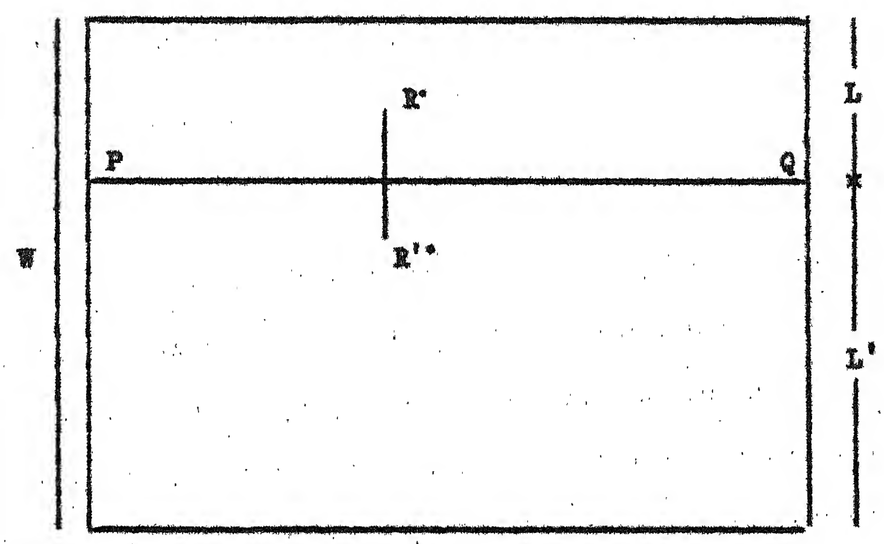
where

$$\phi(\xi') = \frac{(P\theta' x'/b)^2}{1 + (P\theta' x'/b) + (P\theta' x'/b)^2}$$

when advertising is going on simultaneously in favour of both commodities, the net response will be the resultant of two opposing potential responses R^* and R'^*.

Response in this two - directional advertising situation will not be simply $(R - R')$, where R and R' are defined by $R = L\phi(\xi)$ and $R' = L'\phi(\xi')$; this would take no account of the interaction due to the fact that the promotors of X are trying to shift the datum line PQ upwards (see figure 14), while at the same time the promotors of X are attempting to move PQ downwards. Also, in spite of the two-directional attempts to shift PQ, response must be measured from the old line PQ.

<u>Figure 14</u>



Suppose X' promoters have got in a long time ahead and have established a response R' before the X-promoters start, then the latter will be confronted by a "regainable" space which is no longer L, but L + R'. Momentarily assuming that R has to be measured from PQ,

$$R = (L + R')\,\phi(\epsilon)$$

Considering the full two directional promotion situation, Zentler and Ryde allowed for the fact that at every moment there are two new potentially regainable spaces L' + R and L + R', and for the fact that response has to be measured from the original dividing line PQ.

When the two potential responses are in equilibrium,

$$R^* = (L + R'^*)\,\phi(\epsilon)$$

$$R'^* = (L' + R^*)\,\phi(\epsilon')$$

and the resultant responses R and R' in favour of X and X' respectively are

$$R = R^* - R'^*$$

$$R' = R'^* - R^* .$$

Solving for R^* and R'^*,

$$R^* = \frac{\phi(\epsilon)\,[L + L'\phi(\epsilon')]}{1 - \phi(\epsilon)\,\phi(\epsilon')}$$

$$R'' = \frac{\phi(\xi')\,[L' + L\phi(\xi)]}{1 - \phi(\xi)\,\phi(\xi')}$$

The net response, from the point of view of commodity X is

$$R = R' - R'' = \frac{L\,\phi(\xi)[1 - \phi(\xi')] - L'\phi(\xi')[1 - \phi(\xi)]}{1 - \phi(\xi)\,\phi(\xi')}$$

$$= \phi(\xi, \xi')$$

Introducting time lags and variable ξ and ξ' the arguments already used will apply, with $\phi(\xi,\xi')$ replacing $\phi(\xi)$. The final equation for the net response is then

$$R_t = \alpha \Big\{ \phi(\xi_t, \xi_{t'}) + (1-\alpha)\,\phi(\xi_{t-1}, \xi_{t-1}) + \cdots + (1-\alpha)^{t-1}\,\phi(\xi_1, \xi_1') \Big\}$$

Defining $\rho_t = \dfrac{R_t}{W}$, and

$$\phi(\xi,\xi') = \frac{\theta'\phi(\xi)\,[1 - \phi(\xi')] - \theta\phi(\xi')\,[1 - \phi(\xi)]}{1 - \phi(\xi)\,\phi(\xi')} ,$$

$$\rho_t = \alpha \Big\{ \phi(\xi_t, \xi_{t'}) + (1 - \alpha)\,\phi(\xi_{t-1}, \xi_{t-1}') + \cdots\cdots + (1 - \alpha)^{t-1}\,\phi(\xi_1, \xi_1') \Big\}$$

A NUMERICAL COMPUTATION - ZENTLER-RYDE MODEL

The last expression can be replaced by

$$\rho_t = 0.6\,\phi(\xi_t, \xi'_t) + 0.25\,\phi(\xi_{t-1}, \xi'_{t-1}) + 0.15\,\phi(\xi_{t-2}, \xi'_{t-2})$$

which is roughly equivalent to putting $\alpha = 0.6$ and ignoring changes in ξ that took place further back than (t-2).

Zentler and Ryde found that it is not possible to measure θ directly, except in a country in which there is no promotion on either side. But it is possible to measure the percentage of X' in W in a given time period. If the relevant ξ^s and ξ'^s are known, this percentage is a function of θ only and provides a means of determining its value. Thus,

$$\frac{X'}{W} = \theta' - \rho$$

$$= 1 - \left\{0.6\phi\left(\xi_t, \xi'_t\right) + 0.25\phi\left(\xi_{t-1}, \xi'_{t-1}\right) + 0.15\phi\left(\xi_{t-2}, \xi'_{t,2}\right)\right\}$$

$$= 0.6\left\{1 - \phi\left(\xi_t, \xi'_t\right)\right\} + 0.25\left\{1 - \phi\left(\xi_{t-1}, \xi'_{t-1}\right)\right\}$$

$$+ 0.15\left\{1 - \phi\left(\xi_{t-2}, \xi'_{t-2}\right)\right\}$$

$$= 0.6\,G\left(\xi_t, \xi'_t\right) + 0.25\,G\left(\xi_{t-1}, \xi'_{t-1}\right) + 0.15\,G\left(\xi_{t-2}, \xi'_{t-2}\right)$$

where

$$G(\xi,\xi') = \frac{[\theta' + \theta\phi(\xi')][1-\phi(\xi)]}{1-\phi(\xi)\,\phi(\xi')} ,$$

since $\theta + \theta' = 1$, there is only one unknown, namely θ, and, though it is not easy to solve algebraically for θ, the value can be found by graphical interpretation of interpolation.

Once θ is known, $\rho_t = 0.6\,\phi(\xi_t,\xi'_t) + 0.25\,\phi(\xi_{t-1},\xi'_{t-1}) + 0.15\,\phi(\xi_{t-2},\xi'_{t-2})$ can be regarded as a "response curve" showing the effect of varying amounts of advertising in year t in favour of X, taking into account previous advertising expenditure on behalf of X and previous and current expenditure on behalf of X'.

It is this curve which forms the basis of allocation of publicity or advertising expenditure.

"b" has to be assigned a numerical value. This constant is related to β, the real expenditure at the M.A. point, and expresses in practice the unit in which expenditure is measured. Changing the value of b merely changes the scale of all the curves in the same way and so cannot affect their relative positions. But the size of the total budget, S when measured in b units, will depend on the size of b and this constant therefore becomes very important if there is a question of deciding whether or not the budget is large enough to include all countries. In fact b is a scale factor; a change in its numerical value has three main consequences, which are really different aspects of the same thing. If the value of b were reduced then:

(1) For all countries the expenditure corresponding to a particular marginal response would be reduced and, hence, the total budget corresponding to a particular common marginal response would also be reduced. It follows that, for a given budget, a smaller common marginal response is required when b is small and, therefore, that a country whose maximum marginal response is below the common level for a large b will have maximum response greater than the common level if b is made sufficiently small and so will be included in the allocation.

(2) For a given total budget each country is moved further to the right on its own curve.

(3) More countries may be included in the allocation of a given total budget.

If for "exogenous" reasons a certain number of countries have to be included in the allocation, the maximum value of b is thereby determined. In our case we have found that if a slightly smaller value than this limit is taken for b (so as to allow for moderate variations in the budget over the next few years), it would not matter very much if b were to be reduced further, because practically all countries are at a point where their relative positions for optimum allocation would not be changed appreciably, if the present position of each country, on its own curves were pushed further to the right.

SOME FURTHER CONSIDERATIONS OF THE ZENTLER-RYDE MODEL

The response curve has been derived with the only consideration that the ratio in which the total consumption W of X and X' is divided between the two commodities irrespective of any variation in W. In order to predict changes in the absolute consumption of X or X', variations in W must be taken into account. To make the model complete the quantity W (which is here treated as given at any point in time) ought to be replaced by a demand function explaining W in terms of income, prices and, possibly, other variables. Zentler and Ryde found, however, that yearly fluctuations in W within a country were small, compared to differences between countries, and "we therefore thought it justifiable to ignore fluctuations in the W and to treat this quantity as constant for each country in the short run; the values actually used were three year averages"[47].

For practical reasons, Zentler andRyde treated θ as a constant for each country, for prediction purposes, though they recognised that this is, at best, an approximation to the truth; Though θ is not directly affected by changes in W, it may vary from some of the same causes: if, for instance, there is a big difference in price between X and X' then a decrease in total expenditure on X and X' may cause a swing in consumption in favour of the cheaper commodity rather than a proportionate decrease in both, and conversely for an increase in expenditure. But if the two prices are roughly equal θ is not likely to vary from this cause. Such a change in

expenditure could result from a change in income, from a change in the prices of other goods relatively to those of X and X' or, possibly, from a change in promotional activity on behalf of goods other than X or X'. On the other hand, with no change in income, a change in the price of X relatively to X' could also cause a change in θ; here again, θ ought to be replaced by a function which could account for its variations. This could be done given a demand equation for X (in addition to that for W) appropriate to the case of no advertising on either side. But in finding θ by the method described, it is not necessarily assumed that it has in fact remained constant in the past. In effect, what is being found, is how the total demand for X and X' would have been divided between them at a time t if there had been no promotional activity on behalf of either X or X' but all other things had remained the same: in using this value of θ as a basis for prediction, it has been tacitly assumed that θ will remain constant for as far into the future as one wants to predict.

THE ZENTLER-RYDE OPTIMUM ALLOCATION

Given n countries 1,2,........n, with populations N_r and a total budget S to be allocated among them, and given that the per capita response curves for the various countries are

$$R_r = \phi\left(\frac{p_r \theta_r x_r}{b}\right) \qquad r = 1,2,\ldots\ldots,n\ ,$$

the problem is to find the set of x_r that will give the maximum total return for S. That is,

$$\sum_{r=1}^{n} N_r R_r \text{ has to be maximised with the condition } \sum_{r=1}^{n} N_r x_r = S.$$

This requires that

$$\frac{\partial R_r}{\partial x_r} = \lambda \quad \text{for all } r \ ; \quad \frac{\partial^2 R_r}{\partial x_r^2} < 0 \quad \text{for all } r \ .$$

which means that we must choose a set of x_r such that it makes $\Sigma N_r x_r$ equal to S and satisfies the conditions

(i) The marginal responses in all countries are equal.

(ii) The slopes of the marginal response curves are all negative, i.e. all the chosen x_R are to the right of I on their respective curves.

In the particular case under consideration,

$$R = W \frac{k_1 \phi\left(\frac{P\theta x}{b}\right) - k_2}{1 - k_e \phi\left(\frac{P\theta x}{b}\right)} + B$$

where $\phi(z) = \dfrac{z^2}{1 + z + z^2}$, by definition and

$$k_1 = 0.6 \left\{ \theta' [1 - \phi(\xi'_t)] + \theta\phi(\xi'_t) \right\}$$

$$k_2 = 0.6\, \theta \phi(\xi')$$

$$k_3 = \phi(\xi'_t) \ .$$

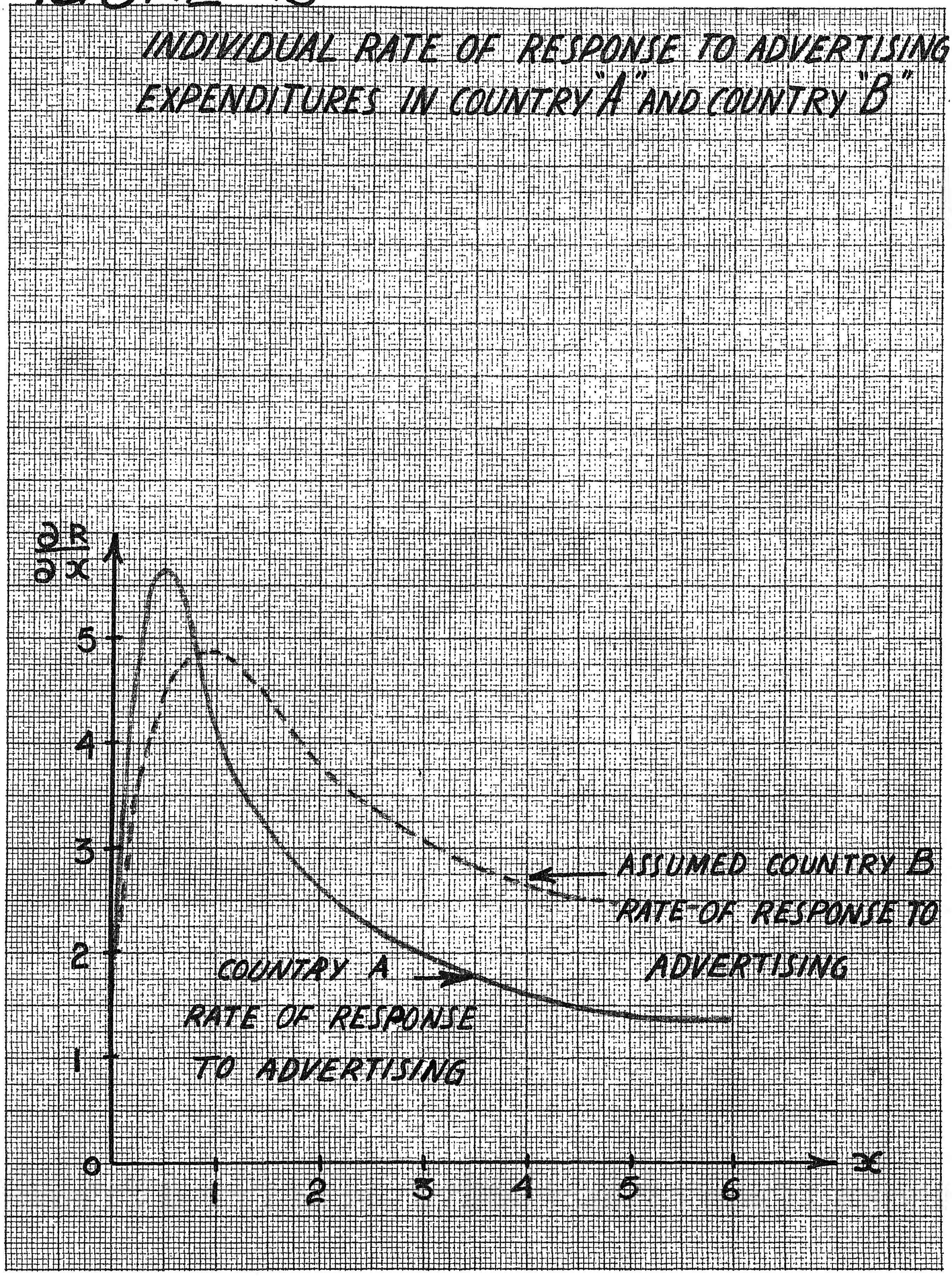
B is a constant involving promotional expenditure in years $(t-1)$ and $(t-2)$. Then,

$$\frac{\partial R}{\partial x} = \frac{WP\theta}{b}(k_1 - k_2k_3)\frac{\phi'\left(\frac{P\theta x}{b}\right)}{\left[1 - k_3\phi\left(\frac{P\theta x}{b}\right)\right]^2}$$

where $\quad \phi'(s) = \dfrac{2s + s^2}{(1 + s + s^2)^2}$

These expressions are for too complicated to make an algebraic solution of the maximisation problem always practicable, but a graphical solution can be found, in cases where the algebraic solution is not practicable, in the following way: The marginal response curve of $\frac{\partial R}{\partial x}$ can be found. Each of these curves will have a maximum at the x corresponding to the point of inflexion in the response curve. To the right of the maximum $\frac{\partial^2 R}{\partial x^2}$ is negative. Then select a series of conveniently spaced values of λ (the common marginal response) and for each one real from the curves the set of x values (one for each country) corresponding to this value of λ and lying to the right of the peak; for each λ form the sum $\Sigma N_r x_r = S'$. Plot the resulting pairs of values of λ and S' in the form of a graph and read from this the value of λ corresponding to the budget available. Lastly, the set of x corresponding to this value of λ is found, which will be the required set.

FIGURE -15-


INDIVIDUAL RATE OF RESPONSE TO ADVERTISING EXPENDITURES IN COUNTRY "A" AND COUNTRY "B"
∂R/∂x
5
4
3
2
1
0
1
2
3
4
5
6
x
ASSUMED COUNTRY B RATE OF RESPONSE TO ADVERTISING
COUNTRY A RATE OF RESPONSE TO ADVERTISING

NUMERICAL ILLUSTRATION OF THE ZENTLER-RYDE MODEL

To illustrate the graphical method of solution suggested by the authors, it will be convenient to consider the case where product X and X' are sold in only two countries. In addition, the following data regarding country A are known:

$W = 100, \quad \theta = .6, \; P = b = 1, \quad k_1 - k_2k_3 = .15, \quad k_3 = .7 .$

Recalling that $\beta = b/\theta$, $1/\beta = .6$, and $(P\theta x)/b = 3/.6 = z$. From these data, values for $\phi(z)$, $\phi'(z)$, and $\frac{\partial R}{\partial x}$ for country A may be easily computed. Table 8 contains the values of these functions for selected levels of per capita advertising expenditures.

The value of $\frac{\partial R}{\partial x}$ for $x = .05$ can be computed as follows :

$$\phi(z) = \frac{(.084)^2}{1 + .084 + (.084)^2} = .0064 ,$$

$$\phi'(z) = \frac{2(.084) + (.084)^2}{[1 + .084 + (.084)^2]^2} = .147 ,$$

and

$$\frac{\partial R}{\partial x} = \frac{9(.147)}{[1 - .7(.0064)]^2} = 1.38$$

The values of $\frac{\partial R}{\partial x}$ in Table 8 are displayed in figure 15.

Figure 15 - Individual rates of response to advertising expenditures in Country A and Country B.

TABLE 8

Individual Rates of Response for various per capita advertising expenditures in Country A

x	s	$\phi(s)$	$\phi'(s)$	$\partial R/\partial x$
.05	.084	.0064	.147	1.38
.15	.250	.047	.326	3.12
.20	.334	.078	.373	3.76
.40	.668	.212	.399	4.95
.60	1.002	.334	.333	5.42
1.00	1.670	.511	.206	4.48
1.50	2.505	.642	.118	3.46
2.00	3.340	.720	.074	2.64
3.00	5.010	.807	.036	1.71
5.00	8.350	.882	.014	0.86

From figure 8 one may read the expenditures per capita in each country that are associated with a common rate of response. By multiplying the per capita expenditures for a country by the population of that country, the promotional expenditure required to achieve a given rate of response is obtained. By assuming that country A's population is N_A = 10,000,000 and country B's population is N_B = 1,000,000 and summing the promotional expenditures for both countries for selected common rates of responses table 9 may be obtained.

By using table 9, management may now select the correct allocation to country A and country B by locating the number in the

total advertising expenditure column that corresponds to the total amount budgeted to promotional activities and reading the allocation for the two countries from columns $N_A x_A$ and $N_B x_B$ on the same line. If a firm's total budget for advertising activity in the two countries is 12 million, approximately 10.5 million should be spent in country A and 1.5 million should be spent in country B. This allocation would insure the highest rate of response for a budget of the size indicated.

TABLE 9

Total Advertising Expenditures in Country A and country B for various common rates of response

Common Rate of Response λ	Per capita advertising expenditure Country A x_A	Per Capita advertising expenditure Country B x_B	Advertising expenditure Country A $N_A x_A$ (millions)	Advertising expenditure Country B $N_B x_B$ (millions)	Total Advertising expenditures $N_A x_A + N_B x_B$ (Millions)
5.0	.90	1.00	9.0	1.00	10.00
4.5	1.05	1.55	10.5	1.55	12.05
4.0	1.15	1.90	11.5	1.90	13.40
3.5	1.50	2.30	15.0	2.30	17.30
3.0	1.75	2.80	17.5	2.80	20.30
2.5	2.10	3.20	21.0	3.20	24.20
2.0	2.65	4.10	26.5	4.10	30.60
1.5	3.35	5.30	33.5	5.30	38.80
1.0	4.50	6.90	45.0	6.90	51.90

The Zentler-Ryde model is equally applicable among different markets or market areas within one country.

Lawrence Friedman[48] formulated fine game theory models for the allocation of Advertising Expenditures among competing firms, but the large number of simplifying assumptions render them of theoretical interest only. The fine models have been developed on the assumption that the major factor governing advertising allocation is competitive expenditure. These models throw no light on the actual effects of advertising on sales. Friedman borrowed heavily from the techniques of other workers[49,50,51,52,53,54].

John F. Magee[55] studied the effect of a particular type of promotional effort on sales, the promotional effort being calls made by a Company's salesmen on retail outlets for the purpose of setting up displays and point-of-sales materials and generally acting as "missionary" men.

OPTIMAL ADVERTISING SCHEDULE OF THE VIDALE-WOLFE MODEL

Suresh P. Sethi obtained the optimal advertising schedule (optimal control) of the Vidale-Wolfe model. Most of Sethi's work was carried out at the department of Operations Research, Stanford University, during 1972.

Suresh Sethi[56] formulated the Vidale-Wolfe model thus: The rate of sales of a product depend on two effects: response to advertising that acts (via the response contant ρ) on the unsold

portion of the market, and loss due to forgetting that acts (via the decay constant k) on the unsold portion of the market.

$$\dot{x} = \rho u(1 - x) - Kx, \quad x(o) = x_o ,$$

where x is the market share (i.e. the rate of sales expressed as a fraction of the market potential or saturation level) and u is the rate of advertising expenditure (a control variable) at time t.

Sethi obtained the optimal advertising schedule (optimal control) of the Vidale-Wolfe model, in the sense that it must maximise a certain objective function over horizon T while attaining a terminal market share x_T within specified limits. Specifically, with the maximum sales revenuw potential π (this assumes a constant margin per unit product), the maximum allowable rate of advertising expenditures Q, and the discount rate i, the optimal control problem is

$$\max_{Q \geq u(t) \geq 0} \left\{ J = \int_0^T [\pi x(t) - u(t)] \exp[-it] \, dt \right\} ,$$

subject to

$$\dot{x} = \rho u(1-x) - Kx , \quad x(o) = x_o ,$$

and the terminal constraint $x(T) \in [x_T^1 , x_T^2]$.

The problem is known as a fixed-end-point problem if $x_T^1 = x_T^2 = x_T$, and a free-end-point problem if $x_T^1 = 0$, $x_T^2 = 1$. The requirement $o \leq x(t) \leq 1$ for all t is automatically satisfied if $o \leq x_o \leq 1$, x_o being the initial market share.

The fixed-end-point problem may be conveniently solved by using Green's Theorem, if there is only one state variable and where the control u appears linearly in both the state equation and the objective function[57]. It is easier to solve this case when $Q = \infty$ (i.e. no upper limit on the advertising rate).

SOLUTION BY GREEN'S THEOREM

From $\dot{x} = \rho u(1-x) - Kx$, $u\,dt = (dx + Kx\,dt)/\rho(1-x)$ is derived. This is substituted in the objective function to obtain the line integral along any curve Γ in (t,x) space

$$J_\Gamma = \int_\Gamma \left\{ [\pi x - Kx/\rho(1-x)] \exp[-it]\,dt - [1/\rho(1-x)] \exp[-it]\,dx \right\}$$

For a simple closed curve Γ, one can use Green's Theorem to express this line integral over the area R bounded by Γ :

$$J_\Gamma = \iint_R \left\{ (\partial/\partial t)\left[-\exp[-it]/\rho(1-x)\right] - (\partial/\partial x)\left[\pi x - Kx/\rho(1-x)\right]\exp[-it]\right\} dt\,dx$$

$$= \left(\frac{1}{\rho}\right) \iint_R \left[\frac{K}{(1-x)^2} + \frac{i}{(1-x)} - \pi\rho\right] \exp[-it]\,dt\,dx\,.$$

For specifying the optimum advertising schedule, the (t,x) space is partitioned into regions where the integrand takes positive

and negative values. For this, the integrand is equated to zero, and the resulting quadratic in $(1 - x)^{-1}$ is solved to give

$$x = 1 - 2K/(\pm\sqrt{i^2 + 4\rho\pi} - i) \ .$$

One is interested in the values of x between 0 and 1, and hence, one may define

$$x^s = \max\left[1 - 2K/(\sqrt{i^2 + 4\pi\rho K} - i),\ 0\right],$$

and the corresponding control

$$u^s = Kx^s/\rho(1 - x^s) = \left(\sqrt{i^2 + 4\pi\rho} - i - 2K\right)\Big/2\rho \ ,$$

which will maintain the state at x^s by using $\dot{x} = \rho u(1 - x) - Kx$, $x(0) = x_0$. The subscript s refers to 'singular arc' for reasons that will become clear later. If we define $\alpha = (\pi\rho)/(K + i)$, it is an easy thing to show that $x^s = 0,\ u^s = 0 \Longleftrightarrow \alpha \leqslant 1$.

Since there is no upper limit on the control u, we will require an important concept of impulse control before we can specify the optimal control - it is an intense advertising campaign for a very short time, an 'advertising pulse'. This concept is made clear in the following lemma.

"Let $P(a,b,t)$ denote the magnitude of an advertising pulse necessary to increase the market share $x(t) = a$ at time t to $x(t^+) = b$ at time t^+ (just after t); then

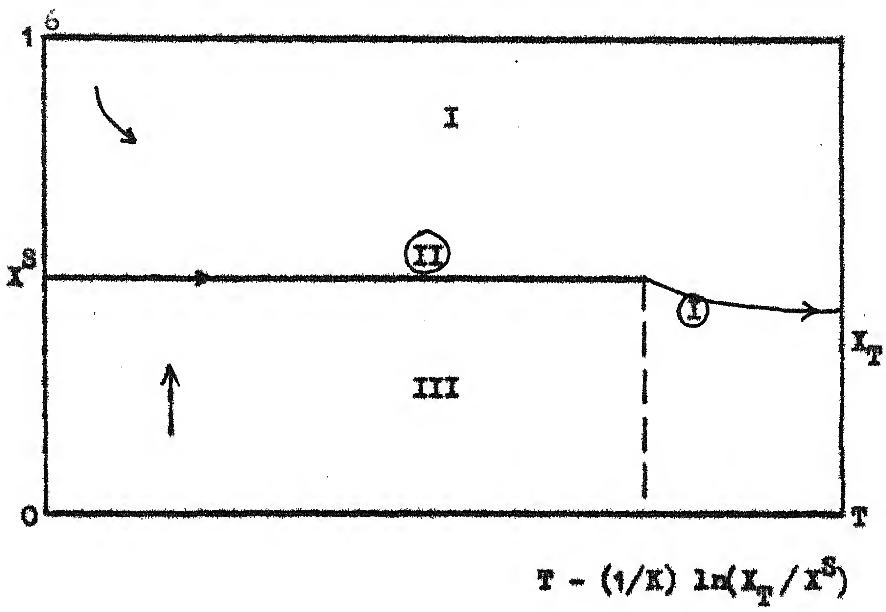
$$P(a,b,t) = \left(\frac{1}{\beta}\right) \ln \left[\frac{(1-a)}{(1-b)}\right] = P(a,b)$$

for all $0 \leq a \leq b \leq 1$ and for all t. Furthermore if $a \leq c \leq b$, then $P(a,b,t) = P(a,c,t) + P(c,b,t)$."[58]

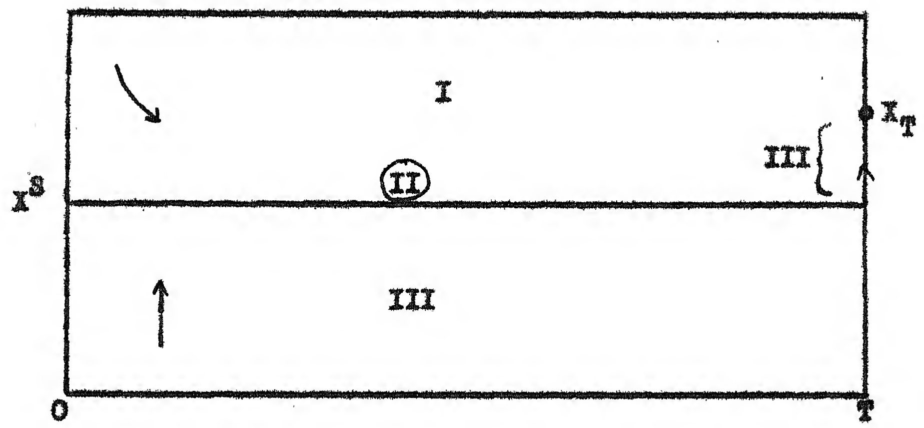
The Optimal Control is specified in the following theorem: "With respect to the Switching diagrams of fig. 16, $\overset{*}{u}(I) = 0$, $\overset{*}{u}(II) = u^s$, and $\overset{*}{P}(x, ,t) = \sup_{y \in III} P(x,y,t)$ for $(t,x) \in III$ is a unique optimal feedback policy"[59].

Figure 16 - Switching Diagrams

(a) $x_T < x^s$



(b) $x_T \geq x^s$



(i) Sasieni has used dynamic programming[60] to characterise the optimal policy for a class of problems that includes the Vidale-Wolfe advertising model. Sasieni's results are not in the closed form, but his characterization is similar to that of Sethi.

(ii) The optimal path between any two fixed points (t_1, x_1) and (t_2, x_2) is the feasible path $\dot{x}(t)$, $t \in [t_1, t_2]$, such that, for all feasible paths $x(t)$ between these points, the areas enclosed between $\dot{x}(t)$ and $x(t)$ lie away from the singular arc with respect to $\dot{x}(t)$. It implies that $\dot{x}(t)$ is the 'nearest' feasible path to the singular arc x^s in the sense that $|\dot{x}(t) - x^s| \leq |x(t) - x^s|$ for each $t \in [t_1, t_2]$. From this remark, it is an easy matter to find optimal controls if the additional constraints specifying the states at intermediate times are present. Furthermore, it is possible to treat additional state inequality constraints of a certain class at

intermediate time intervals.

(iii) It follows that initial and terminal states must be fixed for comparisons between the entire paths. This puts a severe limitation on Green's theorem requiring us to resort to the theory of the maximum principle[61] to solve the general optimal control problem.

(iv) If impulse controls are not allowed, then, in general there does not exist an optimal control for the problem over the remaining class of controls. The reason for disallowing impulse control is its practical infeasibility, since it may not be possible to advertise at a rate higher than a certain Q. With the restriction that the control $u \leq Q$, there is once again a viable optimal control problem, which is treated by switching - point analysis.

For the bounded control problem (i.e. $u \leq Q$), there are cases when $u^s > Q$. In such cases, the singular arc x^s cannot be followed; thus, the criterion of remark (i) does not give a clear-cut choice. Once again, as in remark (ii), the maximum principle has to be resorted to.

APPLICATION OF THE MAXIMUM PRINCIPLE

The current value Hamiltonian[62] is formed as $H = \pi x - u + \nu[\rho u(1 - x) - Kx]$ where the current-value adjoint ν satisfies the differential equation $\dot{\nu} = i\nu + \nu(\rho x + K) - \pi$, or the variable

$\dot{\lambda} = \rho\nu$ satisfies $\dot{\lambda} = i\lambda + \lambda(\rho u + K) - \pi\rho$

The transversality condition on the variable λ for the general problem is $\lambda(T)[x - x(T)] \geqslant 0$, $\quad x \in [x_T^1, x_T^2]$. Clearly $\lambda(T)$ = some constant for the fixed-end-point problem, and $\lambda(T) = 0$ for the free-point problem.

With these definitions, Sethi enunciated the following theorem :

"There exists an optimal control. Furthermore, for $\mathring{u}(t)$ to be an optimal control with corresponding trajectory $\mathring{x}(t)$, it is necessary and sufficient that there exist a nonzero trajectory $\mathring{\lambda}(t)$ satisfying $\dot{\lambda} = i\lambda + \lambda(\rho u + K) - \pi\rho$ and $\lambda(T)[x - x(T)] \geqslant 0$, $x \in [x_T^1, x_T^2]$ and the Hamiltonian maximising condition

$$H\left[\mathring{x}(t),\ \mathring{u}(t),\ \mathring{\lambda}(t)\right] \geqslant H\left[\mathring{x}(t),\ u,\ \mathring{\lambda}(t)\right] \quad u \in [0,Q]$$

for each $t \in [0,T]$ "[63]

The theorems existence and necessity follows from the Pontryagin's maximum principle[64]. Sufficiency follows from the fact that the problem under consideration is a special case of the generalised linear control process[65], for which Lansdowne has proved the sufficiency of the maximum principle.

The Hamiltonian maximising condition in the theorem may be replaced by

$$\dot{u}(t) = \begin{cases} 0, & \text{if } W(t) = -1 + \lambda(t)[1 - x(t)] < 0 \\ [0,Q], & \text{if } W(t) = 0, \\ Q, & \text{if } W(t) > 0, \end{cases}$$

for each t. Here $W(t)$ ($= H_u$) is the coefficient of u in H and, for obvious reasons, it is called the 'Switching function'.

This type of control is known as 'bang-Bang' control in the terminology of optimal control theory. However, interior control is possible on an arc along which $W(t) = 0$. Such an arc is known as a 'singular arc'. More generally, the experimental arcs ($H_u = 0$) on which the matrix H_{uu} is singular are called singular arcs.[66]

Sethi now proceeded to solve the fixed-end-point bounded control problem - for $u \leq Q$, there are cases when $u^s > Q$, and singular arc x^s cannot be followed. When $u^s \leq Q$, Green's Theorem is applicable and will result in a very slight modification in figure 16. However, when $u^s > Q$, a different kind of analysis, known as switching-point analyses[67,68] is needed.

SWITCHING - POINT ANALYSIS

Sethi defined $u^{ss} = \min[u^s, Q]$, with $x^{ss} = \rho u^{ss}/(\rho u^{ss} + t)$ and $\lambda^{ss} = \pi\rho/(1 + K + \rho u^{ss})$ being the associated state and adjoint variables. Furthermore, x^a is defined as $x^a = 1 - (1/\lambda)^{ss}$ and $\beta(Q) = \pi\rho K/(K + 1 + \rho Q)\ x(\rho Q + K)$.

Sethi enunciated the lemmas

Lemma 'a' :- "(i) $\alpha(Q) < 1 \rightarrow u^{ss} = u^{s}$, $x^{ss} = x^{s} = x^{a}$, $\lambda^{ss} = \lambda^{s}$, and $\lambda^{ss}(1 - x^{ss}) = 1$.

(ii) $\beta(Q) > 1 \rightarrow u^{ss} = Q < u^{s}$, $x^{ss} = \rho Q/(\rho Q + K)$ $< x^{s} < x^{a}$,

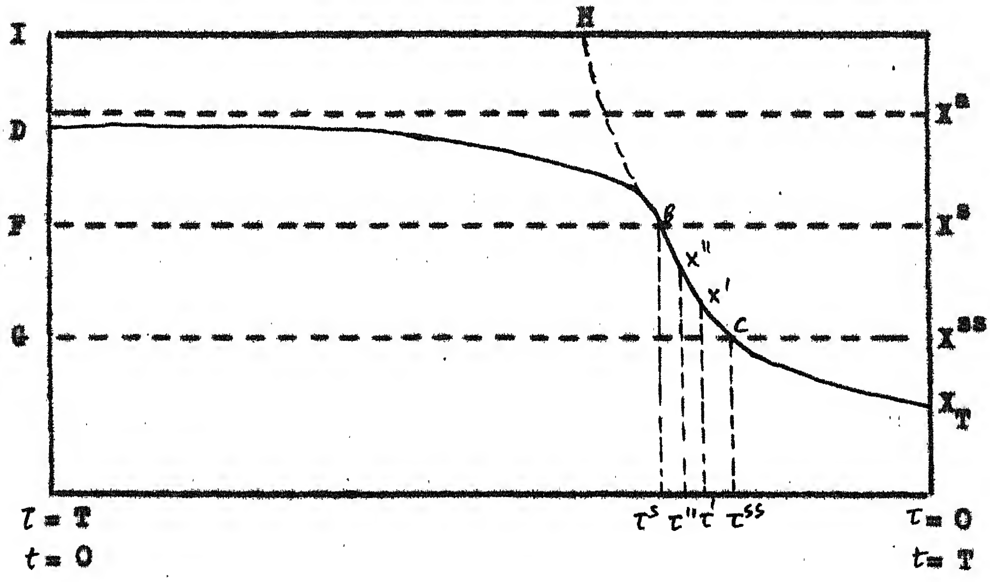
$\lambda^{ss} = \pi\rho/(1 + K + \rho Q) > \lambda^{s}$,

$\lambda^{ss}(1 - x^{ss}) = \beta(Q)$ and $\lambda^{ss}(1 - x^{s}) = 1$."

Lemma 'b' :- The Reverse time-switching function W (T) = $-1 + \lambda'(\ \ )[1 - x'(\ \ )]$ is strictly concave; there exists a unique $(\underline{x}') \in (\ ', \infty)$ such that $W[\ (\underline{x}')] = 0$, $\underline{x}' \in (x^{ss}, x^{s})$. Furthermore, $x^{s} < x'[\ (\underline{x}')] < x^{a}$."

Lemma 'c' :- "Let $\underline{x}'' > \underline{x}'$ and $'' > '$, then (i) $(\underline{x}'') < (\underline{x}')$. and (ii) $x''[\ (\underline{x}'')] < x'[\ (\underline{x}')]$. (see figure 17).

Figure 17



OPTIMAL ADVERTISING SCHEDULE (CONTROL) FOR THE FIXED-END-POINT BOUNDED CONTROL PROBLEM

Figure 18 - The switching diagram for the case $\beta(Q) \leq 1 (u^{ss} = u^{s})$ sub-case (i), $x_T < x^{s} = x^{ss}$.

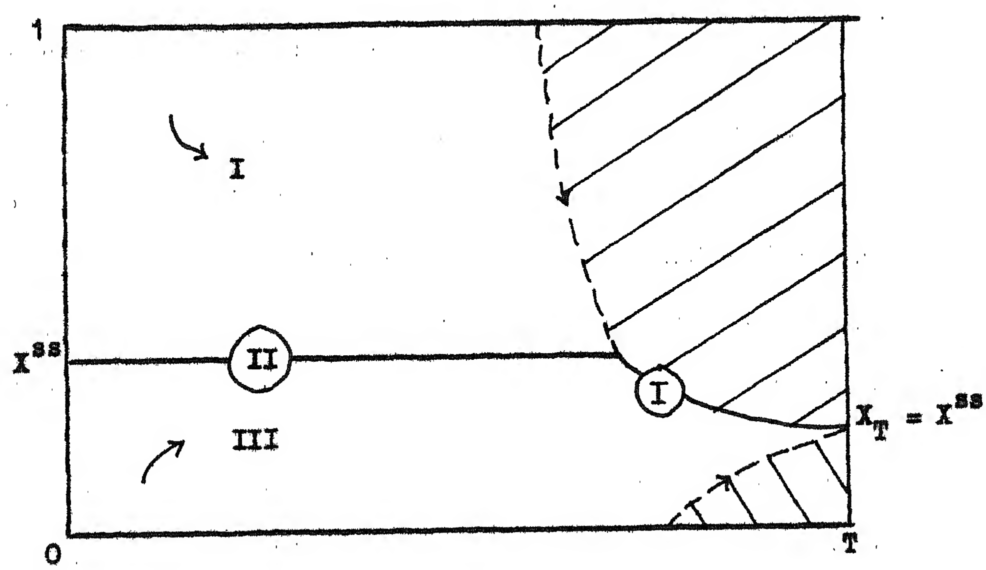


Figure 19 - The switching diagram for the case $\beta(Q) \leq 1$ $(u^{ss} = u^{s})$ sub-case (ii), $x_T = x^{s} = x^{ss}$;

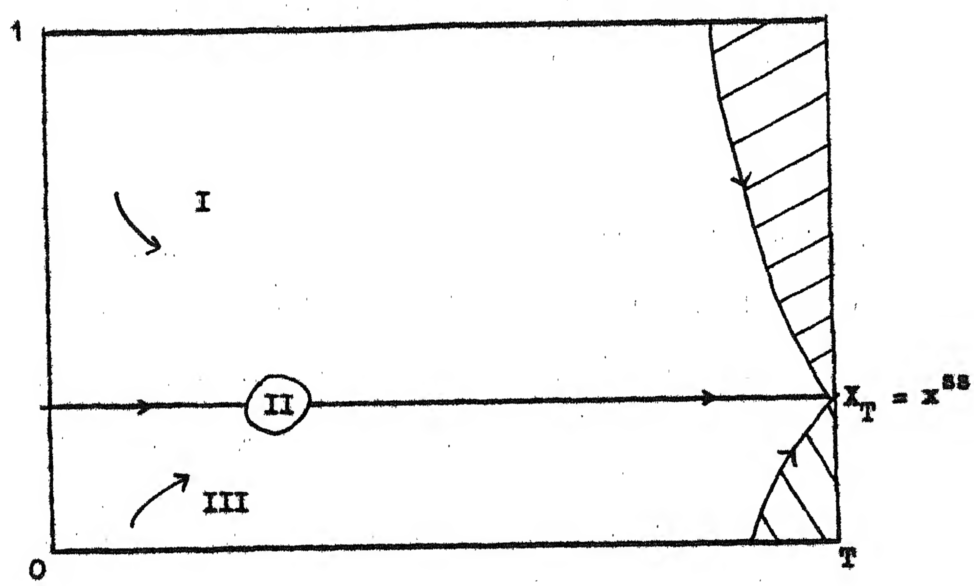


Figure 20 - The switching diagram for the case $\beta(Q) \leq 1 (u^{ss} = u^{s})$, subcase (iii), $x_T > x^{s} = x^{ss}$.

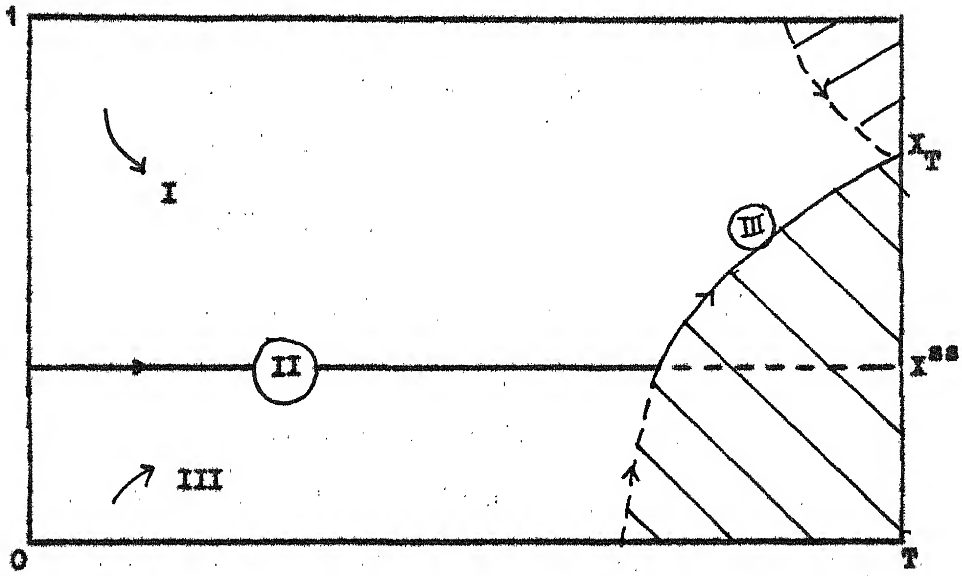


Figure 21 - The Switching diagram for the case $\beta(Q) > 1$ $(u^{ss} = Q < u^{s})$, subcase (i), $x_T \leq x^{ss}$.

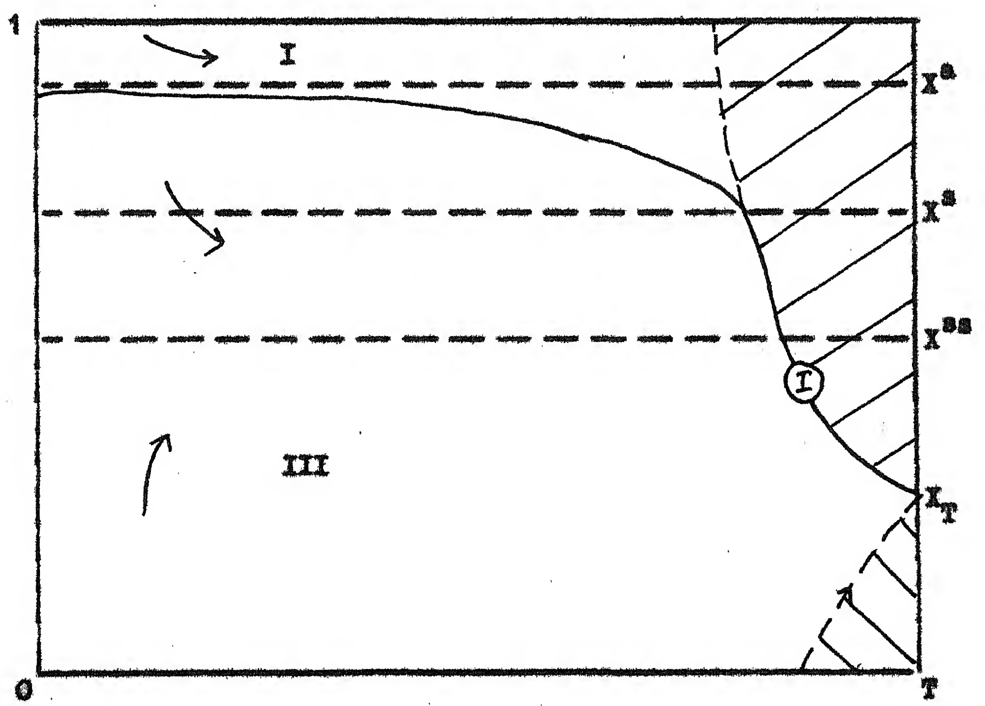


Figure 22 - The Switching diagram for the case $\beta(Q) > 1$ ($u^{ss} = Q < u^{s}$) subcase (ii), $x^{s} > x_T > x^{ss}$.

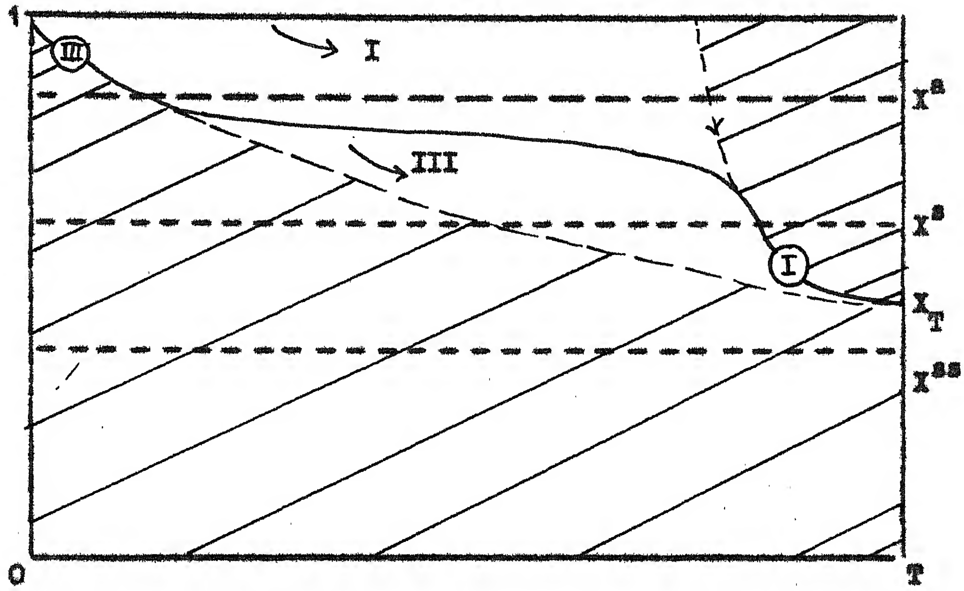
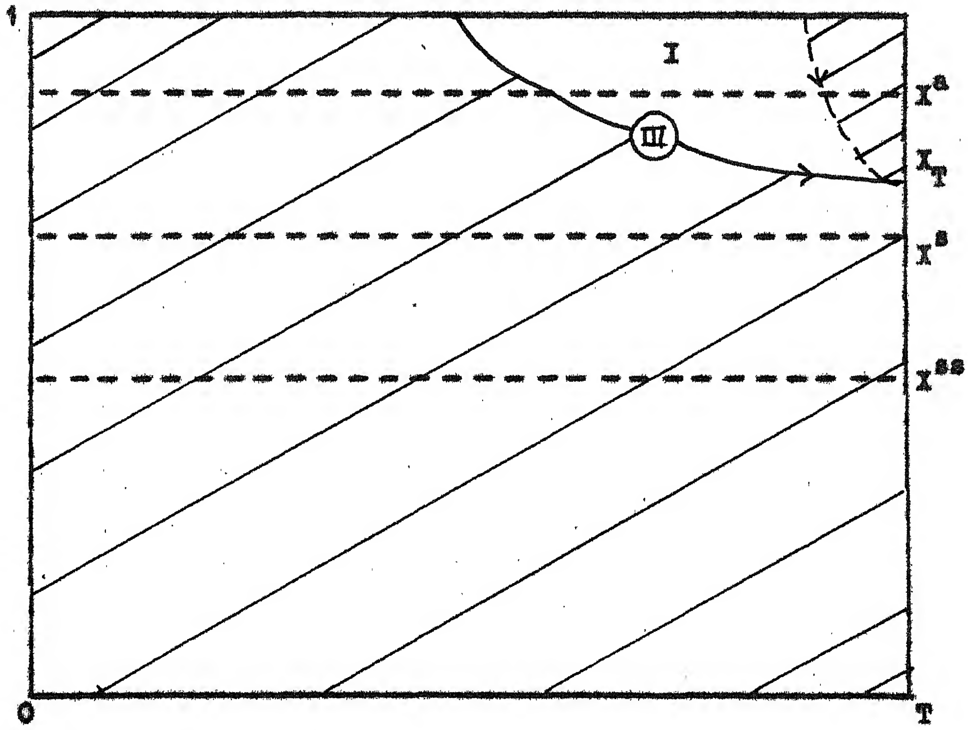


Figure 23 - The Switching diagram for the case $\beta(Q) > 1$ ($u^{ss} = Q < u^{s}$) subcase (iii), $x_T \geq x^{s}$



Sethi postulated the Theorem "with respect to the Switching diagrams of Figures 18, 19, 20, 21, 22 and 23, $\dot{u}(I) = 0$, $\dot{u}(II) = x^s$, $\dot{u}(III) = Q$ is a unique optimal feedback control policy. Furthermore, the Shaded regions are infeasible regions".

OPTIMAL CONTROL FOR THE FREE-END-POINT PROBLEM

Sethi formulated the Theorems :

(a) "For each $x_T \in \left\{0,\ x_T^S = x^S\left[\rho u^s/(K + i + \rho u^s)\right]^{K/K+i}\right\}$, there exists a unique $'(x_T)$ such that (i) $W(\ '(x_T),\ x_T) = 0$ and (ii) $W(\ ,x_T) = 0 \rightarrow \Sigma \geqslant\ '(x_T)$. Furthermore, $'(x_T)$ is a continuous strictly monotonically increasing function of x_T with $'(o) = -\left[1/(K+i)\right] \ln\left[(\pi\rho - K - i)/\pi\rho\right]$ and $\Sigma'(x_T^S) = -\left[1/(K + i)\right] \ln\left[\rho u^s/(K + i + \rho u^s)\right]$. Finally, $\Sigma'(x_T) < -(1/K) \ln x_T$"

(b) "For $\beta(Q) > 1$, $x(t) < x^s \rightarrow \dot{u}(t) = Q$ and $x(t) \geqslant x^s \rightarrow \dot{u}(t) = 0$. When $\beta(Q) \leqslant 1$ $(x^s = x^s)$, we have $x(t) = x^s \rightarrow u(t) = u^s$ in addition. Furthermore, $\left\{x^{ss},\ u^{ss},\ \lambda^{ss}\right\}$ is the unique, optimal long-run stationary equilibrium"[69]

COMPLETE SOLUTION OF THE GENERAL PROBLEM

The following theorem provides the complete solution, "Let $[\bar{x}_T^1, \bar{x}_T^2]$ denote the largest feasible subset of $[x_T^1, x_T^2]$ and let x_T^f be the terminal state in the optimal solution of the free-end-point problem. Then the optimal control $u(x,t)$ can be defined as: (i) If $x_T^f \epsilon [\bar{x}_T^1, x_T^2]$, $\dot{u}(x,t)$ is the solution of the free-end-point problem of the earlier section (ii) otherwise, if $\bar{x}_T^2 < x_T^f$, set $x_T = \bar{x}_T^2$, and, if $\bar{x}_T^1 > x_T^f$, set $x_T = \bar{x}_T^1$; with x_T thus defined, $\dot{u}(x,t)$ is the solution of the fixed-end-point problem obtained in Theorem for optimal control for the free-end-point problem.

SOME REMARKS BY SETHI

The method of Green's theorem is easily adaptable to intermediate state and control constraints. A situation of this sort would arise if management wanted a stage-wise build-up in its market share.

Other easy extensions include time-and state-dependent parameters. Extension to stochastic parameters is straightforward in case of π and i. In most of these cases, the singular arc will be a time function.

Since K and ρ appear in the state equations, the Green's Theorem approach breaks down if they are stochastic. Adequate theory is not available in cases where state-dependent white noise is present in the dynamics and/or in the market-share measurement.

Sethi has discussed these extensions at some length[70], including the situation of institutional advertising, where x is a vector and u a scalar. The analysis reveals the existence of a singular surface. The point of entry to this surface determines the particular singular arc that the optimal control must follow. For more complicated extensions, these singular surfaces may contain 'neighbouring singular arcs', a concept analogous to the Jacobi condition (the no-conjugate-point condition) of the calculus of variations. Very recently, there is considerable theoretical interest in this area.[71]

It should be noted that the optimal control for the Vidale-Wolfe advertising model has the turnpike property[72], the turnpike being the singular arc, in the cases where it is feasible to ride along the turnpike. The optimisation control is the feasible control that spends maximum time along the turnpike, i.e., it uses fastest entry and exit ramps to and from the turnpike.

The managerial implications of the effects of changes in parameters are discussed by Turner and Neuman[73]. In addition, an attempt has been made to enrich the Vidale-Wolfe model by additional behavioural assumptions. The resulting three-state problem is solved using a two-point boundary-value programme.

OTHER RELATED WORK ON ADVERTISING-SALES INTERACTION

PALMER'S WORK

Palmer studied the use of radio as an advertising medium, in

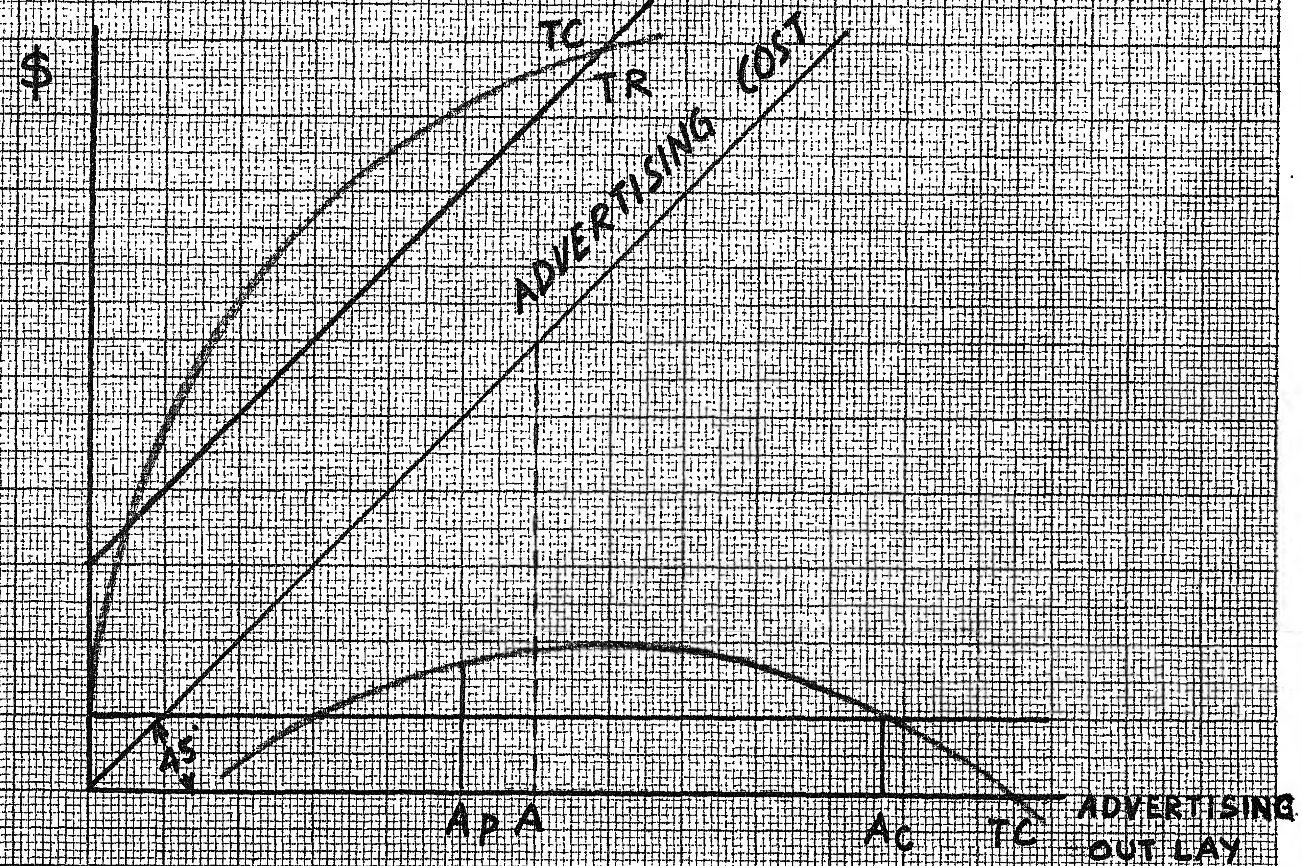
1927, and came to some interesting conclusions. The study was confined to metropolitan Chicago and consisted of 1,000 personal interviews with owners of radio-receiving sets. Palmer concluded "Both the publicity and shopping talk types of radio advertising had directly influenced the purchasing of a considerable number of listeners"[74]

BAUMOL AND SANDMEYER'S COMMENTS ON BAUMOL'S WORK

Baumol in his sales maximisation model assumes " that increased advertising expenditure can always increase physical volume, though, after a point, sharply diminish returns may be expected to set in"[75]. Baumol goes on to say that "unlike a price reduction, a ceteris paribus rise in advertising expenditure involves no change in the market value of the item sold". "Therefore, an increase in advertising outlay must necessarily increase total revenue". "As a result, it will always pay the sales maximizer to increase his advertising outlay until he is stopped by the profit constraint - until profits have been reduced to the minimum acceptable level". Baumol does not consider the case where the firm changes price along withthe advertising budget, both of which influence output, in order to obtain the highest sales revenue given the restriction of earning a minimum profit.[76]

Baumol makes the point initially that the large firm (oligopolist) does have an independent and definite price policy, then abandons it in favour of changing advertising budgets to describe the constrained output equilibrium; However, Baumol does not show how

FIGURE - 24 - ADVERTISING OUTLAY vs TOTAL REVENUE



the price is reached which brings about, along with a given advertising budget, this constrained output equilibrium. Once the constrained output equilibrium is achieved, he returns to the assumption that the firm has a definite price policy: e.g. in criticising Prof. Galbraith's oligopoly model, he says, To Professor Galbraith, as to many investigators, a prime characteristic of the oligopolistic firm is that it does not normally set a price which maximises profits. So far I am in agreement, but in his analysis there seems to be no alternative explanation of the price-setting process"[77]. This statement implies that Baumol has explained the price-setting process, along with determination of the advertising budget, which, as Sandmeyer points out,[78] is not the case.

SEMON'S WORK ON COFFIN'S ASSUMPTIONS

Thomas E. Coffin discussed an experiment in assessing Advertising Effectiveness which will be described in the next section. The assumptions should be scrutinised first, and here Semon has done some original analytical work[79]. Two of Coffin's basic assumptions have been scrutinised (i) Readers and viewers were defined as "those who claimed to have read one or more of the last

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Figure 24 - Advertising Outlay vs Total Revenue

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four weekly issues (of magazine A) or watched one or more of the last four broadcasts"[80] The definition is clear, and the use of the advertising vehicle, rather than the advertisment itself as the exposure criterion is sound. Semon questions the correctness of the assumption - that exposure to one or more of the last four showings of Television programs is equivalent to one or more instances of reading in one or more of the last four issues of a weekly magazine. Although, this equivalence is not explicitly stated in the article, it is implicit in that the analysis presented would be pointless without this assumption of equivalence.

The assumption of equivalence is not necessarily wrong, but it has not been proved. If the analysis presented by Coffin is to serve as a model for practical application, equivalence should be established on a cost basis.

Coffin's study comprised two carefully planned waves of interviews with the same persons, three months apart. The reported media exposure and buying levels were compared between the two waves for each person, and the person classified accordingly. For instance, a person who reported exposure to Television in the second wave, but not in the first, were classified as having "started television".

(COFFIN'S WORK)

(Coffin's experiment was a 2-wave panel study, in which the same individuals were reinterviewed at two points in time three months apart)

Coffin draws a close parallel between such exposure - defined groups and possible advertising strategies. Although the analogies are plausible, their accuracy is doubtful in terms of the exposure definition. This definition implies a process whose occurence or nonoccurence within a four week period constitutes a significant dichotomy.

The results reported fail to bear out this implication. If the occurence or nonoccurence within a four week period constituted a significant dichotomy, it should be associated with differences in buying levels that are clearly greater than buying-level changes attributed to unspecified, general change in degree of exposure.

Changes in buying levels were shown according to two exposure change criteria. In table 2 of the Coffin article, the criterion was increased or decreased in sales.

Coffin's sample was a prelisted probability sample, representing a typical medium-sized midwestern market; 91 / of the original respondents were recovered on wave II. The respondents were male and female household heads, with a final sample size of 2, 441. The study covered 22 brands advertised both on network TV programs and in weekly magazine A, representing eleven different product categories : Beer, Canned soup, cigarettes, Gasoline, Headache remedies, deodorants, Home permanents, packaged cheese, packaged desserts, Razor blades, toothpaste. For each of these categories the respondent was questioned as to what brands he or she had "personally purchased in the last four weeks". The "buying figures"

to follow, therefore, represent the percentage of the total sample (or specified sub-group) who claimed to have purchased the advertised brand in the last four weeks.

Table 1 - Coffin

Some relationships of buying to overall Advertising Exposure

1. Overall Results of Survey

	/ buying in past 4 weeks
Total sample, wave I	19.4 /
Total sample, wave II	19.6 /

2. Relationship of buying to advertising exposure

	/ buying in wave II
Exposed to advertising (TV and/or magazine)	20.5 /
Not exposed to advertising	16.9 /

3. Relationship of buying to degree of exposure

	/ buying in wave II
Not exposed to advertising (TV and/or magazine)	16.9 /
1 unit of exposure (TV and/or magazine)	18.9 /
2 units of exposure (TV and/or magazine)	20.1 /
3 units of exposure(TV and/or magazine)	21.8 /
4 units of exposure(TV and/or magazine)	24.0 /

Coffin's Table 1 shows that if we break out those who are exposed to advertising for the specified brands (any type, either

TV or magazine) as compared with those not exposed, we note an appreciable difference in their buying levels on the second wave : 20.5 / versus 16.9 / . If we analyse the exposed group by degree of exposure, we begin to see a trend. The more units of advertising a group is exposed to, the higher its buying level on wave II (where exposure to one medium at one period equals one unit). By wave II, there was a 42 / spread between the levels of the least-exposed and most exposed groups.

Coffin subjected the panel data to "dynamic analysis", the study of change. The possibility of detecting significant relationships is enhanced when people are observed in the process of change. An analysis of respondents in terms of change in their advertising exposure revealed that changes in exposure were associated with changes in buying. Respondents who experienced an increase in exposure (to either form of advertising,) showed an increased level of buying; with no change in exposure, there was no change in buying level; and with decreased exposure came decreased buying, as shown by Coffin's Table 2.

<u>table 2 - Coffin</u>

Relationship of buying to changes in Advertising Exposure

	/ Buying		Relative change
	wave I	wave III	
Increased exposure	19.6	21.0	+ 7 /
No change in exposure	20.3	20.4	0
Decreased exposure	21.0	19.4	- 8 /

The increased exposure group consisted of those respondents who reported more units of exposure onthe second interview than on the first. The no-change group were those who reported the same numbers of units both times. The decreased exposure group were those who reported more units on wave I than on wave II; thus at the time they were most heavily exposed, their buying level was higher, and when they became less exposed, their buying level became lower.

Dynamic analysis may be applied to the dependent as well as the independent variable. Examination of the "buying dynamics" enables us not only to measure changes in levels, but to perceive the mechanism whereby these changes come about. Those who were buying on wave I can be divided into two components. Some were still buying the same brand on wave II three months later. They may be termed the "continue buying group". Others reported not buying this brand on wave II; these are the "stop buying" group.

Likewise, among those who were not buying the brand on wave I, some will be found buying it on wave II (2start buying"); and others are still not buying it ("non buyers").

These buying dynamics are related to changes in advertising exposure, as shown by Coffin's Table 3.

Table 3 - Coffin

Buying Dynamics and Changes in Exposure

	/ stop buying	/ continue buying	/ start buying
Increased exposure	8.0	11.6	9.4
No change in exposure	8.1	12.2	8.2
Decreased exposure	9.4	11.6	7.8

In Coffin's Table 4, buying levels were analysed according to reported "starting, maintaining, or stopping" of exposure in the two media separately.

Table 4 - Coffin

Changes in buying level related to changes in exposure

Exposure change	Buying levels		
	wave I	wave II	change
Increased exposure units (combined)	19.6 /	21.0 /	+1.4 /
"started" viewing	18.7 /	20.6 /	+1.9 /
"started" reading	20.5 /	21.0 /	+0.5 /
Average "started"[a]	19.6 /	20.8 /	+1.2 /
Decreased exposure units (combined)	21.0 /	19.4 /	-1.6 /
"stopped" viewing	21.6 /	19.2 /	-2.4 /
"stopped" reading	19.9 /	19.4 /	-0.5 /
Average "stopped"[a]	20.7 /	19.3 /	-1.4 /

a = these averages assume that equal numbers of persons "started" (or "stopped") reading and viewing. The averages are probably within ±.3 of the figures shown.

Coffin's Table 4 shows that "starting" or "stopping" to read or view is not associated with changes in buying levels that are substantially greater than those noted in connection with general unspecified, changes in the overall level of exposure.

In the TV viewing question, a list of programs was shown and the respondent was questioned about his personal viewing of each in the last four weeks. In the case of weekly magazines, a list was also used, with the questioning directed toward the last four weekly issues or watched. One or more of the last four broadcasts. Thus, in the preceding sections the unit of exposure is the advertising vehicle, rather than the commercial or advertisement.

To maintain comparability, the same questionnaire wordings and sequences were employed in both, waves of the study. To minimise any possibility of the buying responses being influenced by the media questions, these topics were separated as widely as possible, so as to obtain the information about buying before asking about media, exposure, with a number of other questions intervening.

Moreover, several additional products, programs, and magazines were included in the lists, in order to divert attention further from the questions of special interest and to lessen the chance of

producing a spurious relationship between a particular product and a program or magazine. Careful analyses of previous studies have shown that such precautions have been effective in minimizing the possibility of inadvertent "reversal" of cause and effect or of " conditioning" the responses on wave II by prior questioning on wave I (Reference NBC Studies[81,82]).

The figures reported in Coffin's Tables 1,2,3 and 4 represent the unweighted averages of the twenty-two brands studies. Inspection of the individual brands indicates that these averages properly reflected the experience of the majority of the individual brands; about 75 / of the individual brands showed the same patterns as those pictured by the averages, while 25 / of the brands deviated from the average pattern in one way or the other.

As shown in Table 1, the first overall finding of the study of was that, on a total sample basis, there was some difference between the buying levels for wave I and wave II. This pattern is true regardless of medium.

Semon's conclusions from Coffins experiments were paraphrased thus: "Two tentative, related conclusions can be drawn (i) rather exposure turnover is lest important than has been assumed, or the criterion of reported exposure within the last four weeks is not the most relevant and discriminating criterion available. (ii) By leaving out the alternatives of increasing or decreasing advertising weight, the strategies of "advertising analogs" did not include strategies as important as and more realistic than those presented in the Coffin article".

PALDA'S REVIEW OF LITERATURE

Dr. Kristian S. Palda reviewed the literature on the sales effects of advertising[83] in September 1964.

The Journal of Applied Psychology, 1945, carried several "limited objective" studies (e.g., the influence of color on readership).

The Journal of Industrial Economics, checked by Palda from 1952 to July 1963, contained an article by Maffei (1961), employing hypothetical models of market behavior and hypothetical illustrations of the effects of advertising, avoided the exasperating problems presented by real data.

The Harvard Business Review, since 1922, has carried three articles appraising the sales effects of the total advertising effort. Vaile (1927) studied the variations in advertising lineage and sales of more than 200 companies during one business cycle (1920-1924). Tousley (1944) paid attention, among other things, to changes in unloads of Washington State apples in relation to fluctuations in advertising expenditures (expressed in dollars per 100 inhabitants of 38 cities). Hollander (1949) described an investigation of the influence of advertising on the sales of a "nationally distributed" drug product. He did not outline clearly the graphic multiple correlation analysis he used, but claimed he could determine the cumulative advertising effect.

The Journal of Advertising Research in its first three volumes

(ending with December 1963) carried four articles reporting on studies linking sales to total advertising effort. Meissner (1961) made a cross-sectional, multiple regression study of the effects of advertising on sales of lettuce. Bankds (1961) touched incidentally upon the net influence of advertising on sales of coffee and cleansers. Henderson, Hind and Brown (1961) reported on an ingeneously designed, large-scale experiment which used variance and covariance analysis to evaluate the sales effectiveness of two campaign themes. However, their article does not examine the total advertising effort.

Becknell and McIssac (1963) performed an experiment using a factorial design, in which the level of advertising effort, as measured by counting TV commercial minutes, was related to sales, as measured by purchase reports in telephone interviews with housewives. Hoofnagle (1963) dealt with increasingly complex analysis of advertising effectiveness. First, during a promotion campaign for lamb, store audits were made before, during and after the promotion. Second, sales of cottage cheese were observed in test and control markets (matched cities). Third, the effectiveness of frozen orange juice advertising was assessed by an unusual analysis of regression residuals. Fourth, the effects of another lamb promotion were probed in a double change-over experiment.

The Journal of Business has carried a number of reports on the measurement of total advertising effect in terms of sales. Cover et al. (1931) made a graphic analysis of the relationship between

fluctuations in department stores sales and their newspaper lineage (which was taken as the best available indicator of advertising outlays). Schoenberg (1963) conducted a pioneering study of cigarette demand in which per capita yearly consumption of cigarettes was regressed on "real" price of a thousand cigarettes, on the amounts spent by the four leading cigarette companies on newspaper advertising, and on time. Though primarily interested in the price-quantity relationship, Schoenberg carefully analysed advertising's influence as well. Cowan (1936) contributed an ingenious cross-sectional study of the relationship between regional circulation figures of the Saturday Evening Post (which carried Chevrolet advertisements) and regional Chevrolet registrations. His cross-tabulation method yielded among other things, data suitable for marginal analysis. Brown and Mancina (1940) tested the hypothesis that the relationship between the sales and advertising expenditures of 108 department stores could be expressed by a linear function. Analysis of (residual) variance did not yield conclusive results. Roberts (1947) published an article which remains a classic in advertising measurement. Using multiple regression techniques, he analysed consumer panel data on two rival pharmaceutical brands. He succeeded, among other things, in isolating the net influence of one manufacturers advertising on another firm's sales. Tull (1955) examined the sales decline of Sapolic soap and concluded that advertising outlays were not to blame. He did not employ statistical techniques.

The Journal of Farm Economics (checked by Palda from the 1930

issues onwards) published, in 1961, an article by Nerlove and Waugh which described the use of an ingenious econometric approach in estimating the short and long-run effects of advertising for Florida oranges.

The Journal of Marketing, since its appearance in 1936, has carried four articles reporting on empirical research into the link between total advertising effort and sales. Wagner (1941) investigated the effects of industry-wise advertising (cigarettes, autos, department stores) on sales against the background of a business cycle. Root and Welch (1942) used consumer panel purchase data to evaluate the effectiveness of radio programs and media. Berreman (1943) examined the simple correlation between indices of advertising expenditure and sales of individual novels, but on the whole failed to uncover a relationship between the two. Dickens (1955) reported a simple before-and-after experiment in which it was found that the use of advertisement posters stimulated the sale of milk.

The Journal of Political Economy (checked by Palda from 1930 onwards) recently carried an article by Telser (1962) which is probably the most interesting study of the advertising - sales relationship published in it. Using both absolute and market-share data of certain periods between 1913 and 1960, Telser analyzed the competitive struggle of the major cigarette companies as reflected in advertising and product change. He uncovered, among other things, evidence of overspending on advertising by one company.

Palda also surveyed the work of Vidale and Wolfe (1957). Vidale and Wolfe claimed to have discovered patterns of sales response to advertising outlays that are susceptible to generalisation. Controlled advertising experiments conducted by major industrial concerns with a number of products apparently revealed the existence of such parameters as the Sales Decay Constant, i.e., the rate at which customers are lost when advertising is stopped. According to Palda, the reader is not offered enough information to permit verification.

Palda's survey does not cover such trade periodicals such as "Printers Ink", "Advertising Age" or "Media/Scope", nor does it deal with occasional papers such as those published by the Advertising Research Foundation. [The Advertising Research Foundation has published 32 studies of Advertising - Sales Interaction presented at Advertising Research Foundation Conferences or at the Advertising Research Foundation Operations Research Discussion group].

THE WORK OF MARCELL MARANTZ

Marcell Marantz[84], Special Assistant to French Government Cabinet Secretary in-charge of economic statistics, measured the effectiveness of department- store advertising by noting its impact on the weekly moving average sales curve. Marantz conducted his research in France, over a time period of ten-years. This research had two objectives :- (i) To work out and test an accurate and reliable tool for measuring the effectiveness of advertising (ii)

To develop practical rules for the guidance of management in its advertising policy. The observations, findings, and conclusion relate exclusively to advertising by department stores. There is, however, sufficient evidence, based on thousands of diversified facts, to permit the extension of the findings to all stores.

A superficial observer would measure advertising effectiveness in a departmental store by counting the number of advertised items which have actually been sold following a promotion. This approach is of very little use because it gives an incomplete and misleading picture. (a) It cannot be ascertained that some - even all - of the sold items would not have been bought in the absence of advertisements. To eliminate this assumption it should be noted whether or not sales of similar items did not decrease at the time of promotion - a considerable task both conceptually and in practice. (b) In fact, the store promotional effort is not, in general, to sell a specific item; it is to sell the store. Therefore, the store is sold and promotion is successful when, and only when, there has been a real increment in the total sales of the store or rather in the total sales of the department containing the advertised items.

This concept implies that it would be feasible to evaluate what the sales would have been without advertising, in other words, that it is possible to calculate the "normal" sales.

The Technique -

The sales of a department store vary widely from one day, of

the week to another. For instance, Saturday's sales are usually higher than Wednesday's. Differences between two successive days are not indicative of the actual variance in activity. The real phenomenon is blurred by actual inherent day-to-day variances. The first step is to smooth out the day-to-day variances. Experience points out that a weekly moving average is suitable for this smoothing. The weekly moving average is calculated as follows - letting each letter represent daily sales, and each number represent the date

$M_1 + Tu_2 + W_3 + Th_4 + F_5 + S_6$ = Weekly moving total 1.

$Tu_2 + W_3 + Th_4 + F_5 + S_6 + M_8$ = Weekly moving total 2.

$W_3 + Th_4 + F_5 + S_6 + M_8 + Th_9$ = Weekly moving total 3.

An adjusted sales curve reveals the actual variation which may influence the activity of a department at one precise point (i.e. day). To check this, let us see whether a specific event modified the profile of a smoothed sales curve. Two undebatable examples give the answers. One example revealed the smoothed curve of the umbrella department. Each time it rained, a strong reaction could be observed. During December 1964, a transportation strike reduced the activity of Paris stores. The impact was clearly shown on the smoothed curve.

From these and many other observations, we come to the following conclusions. When a specific cause (or action) affects the sales of a store department, its smoothed sales curve visibly

deviates. Conversely, when some action intended to cause an increase (or, by the same token, a decrease) does not change the course of the smoothed sales curve, this cause has no effect. "Normal" sales (i.e., assumed sales without advertising) can be calculated. Any doubt can be clarified by comparison with a previous year if no advertising (or other cause) was made on the same day. The use of Semilogarithmic paper helps to extend short - term trends. The end of the deviation, hence the return to normal, can always be easily determined taking into account the span of the moving average.

The Procedure -

Direct Effects - Each department concerned with a specific advertisement or promotion, is observed for the appearance of an impact on its smoothed sales curve on the day of the promotion and the days following it.

The impact, if any, is evaluated by reference to the "normal" curve in terms of number of days of the impact, and value of the impact, in money. "Plus" sales constitute the difference between the normal curve and the actual curve.

There is no final plus sales if the duration of the deviation from normal is less than the span of the moving average. This kind of surprising accident is called "compensated sales". Further evaluation examines the ratio of plus sales to normal sales and the ratio of plus sales to the cost of promotion. This set of

measurements gives everything needed to assess effectiveness and to guide management for further action.

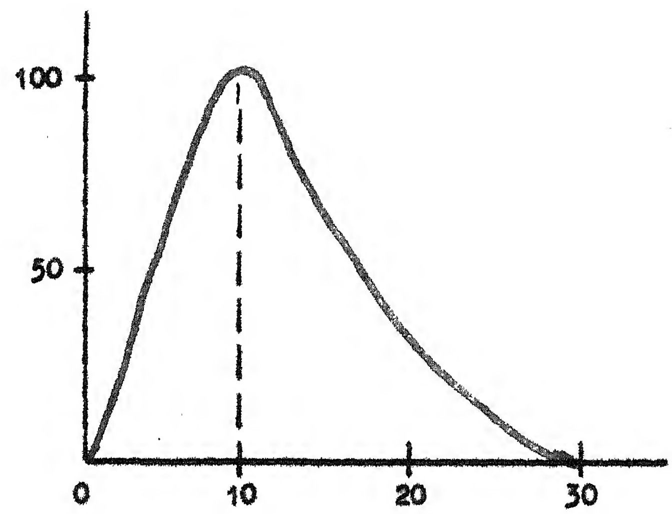
Indirect Effects - Customers induced to come to the store and to buy the advertised item may buy in departments which were not advertised. These indirect effects may be checked and measured by observing the smoothed sales curves of non-promoted departments - on the condition, of course, that no other course (e.g., the weather) has affected the curve at exactly the same time. Those departments canbe selected by their nature (similar to the advertised ones). Incidentally, these measurements give a clue to the problem of complementary (or interrelated) sales. Indirect effects are measured in the same manner as direct effects. They give the same set of parameters for effectiveness. The sum of direct and indirect effects is, of course, the real gross result of a promotion.

THE COUFFIGNAL MODEL

(after M.L. Couffignal, French Mathematician and cybernetician)

The impacted section of a smoothed sales curve can generally be represented by the following mathematical model : $y = t^{\alpha} \times 1^{-Kt}$, t being the number of days of the impacted section of the smoothed curve, y the value of the impacted section, α and β being variable parameters, and 'Ka' fixed coefficient.

Figure 24 - Couffignal's Model



Couffignal's curve shows that "the social phenomenon of a population responding to an advertising stimulus is not linear". However, the shape of the curve is similar to Denis Gabor's expansion curves which correspond to the response of an electrical filter to an impulse or, identically, the response of a nerve to excitation. The maximum is taken as the origin of the ordinates. The characteristics of the formal curve are the following :
(a) There is no inflexion point in the left part (from point 0 to maximum). This means that the sales are increasing more at the very beginning than later. (b) The maximum occurs at one-third of the whole time-length of the impacted section. Accordingly, knowing when the maximum occurs, the end of the impacted section is known.

An investigation can be started along these formal lines. Evaluation of the parameters may lead to a better knowledge - hence

improved management - of promotional action.

Although this investigation may develop in an interesting manner; it has been temporarily discarded.

MAIN FINDINGS OF MARANTZ

Several thousands of observations were analyzed, over a long time - span (three to five years, sometimes longer), concerning stores which differ in size (large or small) and location (various countries such as France, Sweeden, Belgium, Denmark, West Germany, or in the same country, using two separate premises of the same store). Apart from very few exceptional cases, the observations were similar.

Timing an advertisement - Advertising should be timed, whenever possible, at the very beginning of the increasing phase of the demand pattern. [The advertising results are determined by the demand pattern and not the reverse. The demand pattern is the profile of the curve of departmental sales resulting spontaneously from the consumer's demand without any marketing or promotional activity. Demand patterns may be constant, trend (increasing or decreasing), seasonal demand (cyclical) trend-seasonal demands; and end-of-season demand]. By doing so, chances of success are the highest with the lowest cost. The quantity of advertising is a function of (a) the strength of the demand pattern, (b) the content of the promotional action (its message). The greater the strength and the more appealing the message, the less is the

promotional effort that is required.

This finding runs counter to the scale "law": more publicity: more sales. This "law" is definitely not supported by our observations. An equilibrium point exists beyond which incremental advertising expenditures bring little or no return.

Immediate effectiveness of advertising is the most efficient way of obtaining long-term effects.

THREE OPINIONS ON MEASUREMENT METHODOLOGY

Henderson, Brown and Hind[85] are of the opinion that when time and other factors obscure the underlying forces affecting sales, a simple technique can sharpen the picture : covariance analysis"

Rotzoll[86] is of the opinion that, as regards the Starch and Ted Bates correlative measures of Advertising Effectiveness, "Both approaches hold that advertising's contribution to sales is reflected in the higher usage by those who can recall the advertising. Unless proper controls are used, non-advertising factors can account just as well for any such usage differences".

Maurice W. Sasieni[87] is of the opinion that "we can - and should - measure the present and future sales effects of major marketing forces, singly and in combination. The technique : multi-factor experiments".

A FEW OTHER STUDIES

Eldridge[88] believes that, by his suggested after-the-fact appraisal, the ultimate effectiveness of the advertisement in advancing the objectives for which it was designed can be measured.

Ferber and Wales[89] attempted an evaluation of the impact on physicians of pharmaceutical - magazine advertising and direct - mail literature, with regard to the reception accorded this material and the cost of the advertising.

Wallace[90] has given a "method" for determining the contribution made by advertising. Wallace's "method" is as follows :

i) "Make a list of all the working functions of the business (research and development, maintenance, accounting, sales, etc.). But do not include advertising."

ii) "To each one of the listed functions, allocate the exact amount of sales or profit which can properly be credited to that activity."

iii) "Add up the allocations."

iv) "Deduct the sum of these allocations from the known total of sales or profit for the business".

v) "What remains is the contribution to business."

Karstein[91] is of the opinion that, with two possible exceptions, the sales effectiveness of advertising can be measured. The exceptions are : (i) A new product introduction: where no or practically no sales exist, it may be necessary, temporarily, to

measure for instance the awareness created by the campaign. But as soon as the sales are sizable, sales have to be measured. (ii) where advertising represents only a negligible factor in the company's marketing - mix. But in this case the measurement is also not so important as is the effectiveness measurement of those elements which cost substantially greater amounts of money. That is why careful experts in the line frequently qualify their remarks in respect of sales measurability with the word "sometimes".

Karstein's conclusions are as follows : (i) The numerous reasons so often cited as to why the sales effectiveness of advertising cannot be measured do not appear to hold (ii) It is no more difficult to measure sales effectiveness than the other steps of the supposed-to-exist hierarchy of effects of which so frequently measurements are taken. (iii) Just as it insists on sales forecasts, top management should insist on predictions of what exactly the planned advertisements will do to sales and demand greater accuracy from year to year. (iv) Sales Effectiveness has been measured and is being measured 'every working day', (v) Pre-measurements are at least as useful as post-measurements, probably more so. (vi) There should be a budget for effectiveness measurements. If there is none, the relatively small amount should come out of the respective marketing input. (vii) If properly carried out, effectiveness measurements not only can be done but also pay off handsomely.

Taylor[92] gave three reasons for rejecting the assumption that

sales - effectiveness of advertising cannot be measured.

(i) A conceptual framework can be developed to produce measurable promotion objectives.

(ii) The methodology to use in promotional effectiveness measurement is available.

(iii) When the proper conceptual framework has been developed and the proper methodology applied, promotion effectiveness is being measured.

Taylor also gave reasons why sales effectiveness measurements are not made :

(i) Top management has not insisted on getting them

(ii) Top management has been talked out of this demand by those who are responsible for the firms advertising

(iii) The lack of precisely -defined advertising goals has to result in 'compromising, non-sell advertising'- then there is 'nothing to measure'.

(iv) The persons concerned are unable to produce advertising that makes people buy.

Taylor is also of the opinion that the failure of most experiments on promotion effectiveness to produce meaningful results stems from insufficient preparation in two areas : understanding the purchasing process for the product, and appropriate use of experimental design.

The effectiveness of promotions can also be measured under field conditions, although inadequate preparation before the test

may obscure that fact. A thorough understanding of the purchasing process for the product is mandatory before the test begins and rigorous adherence to experimental design during the test is necessary, if meaningful results are to be achieved.

Taylor cites three good reasons for concluding that sales - effectiveness can be measured, as given above.

Twedt[93] makes a general observation on the preconditions for successful advertising. "When four conditions are right- i.e. when clear, definite objectives are set in advance; when a real reason for buying is presented; when the message is directed to qualified prospects; and when exposure is adequate - then advertising has a chance to work. Without these four conditions, the advertising dollar is likely to be spent without appreciable measurable effect on either sales or profits.

Merriman[94] described a control-experimental approach to determine advertising's influence on sales. "While almost all advertising is of some benefit, some is better than the rest".

Merriman studied five advertising appeals. The questions put to research were : which of the five appeals was the most effective? What relative importance should each be given in the various compaigns? Where there any of these appeals that should receive increased emphasis? Should any receive less emphasis?

The sales trend measured in the control markets which received no additional newspaper advertising was considered to represent the

normal seasonal trend. In analysing the results of this test, the sales trend (as shown by the sales audits) in each of the various sets of test or "extra advertising" markets was measured against the control - market trend. In this way, the relative amount of sales in each of the test markets that might be attributed to the additional advertising pressure was determined.

Twedt[95] reported the results of an effectiveness study. During the 1963 baseball season, Oscar Mayer and Co. sponsored 162 Milwaukee Braves Radio broadcasts at a total cost of $ 80,000, with a 38 - station lineup covering 800,000 radio homes. During the last week of the season, 1,200 completed telephone interviews were made 200 in each of six Wisconsin cities. Respondents were asked how many of the games they had heard, who sponsored the games, what brand of Wieners they bought most often, and how many poinds of Wieners they had used in the last 30 days.

Table 1 - Twedt

Comparison of listening and Non-listening households

	Total households	Listening households	Non-listening households
No. of Radio households in station lineup area	800,000	416,000	384,000
Annual Wiener consumption per household	23 lbs.	26 lbs.	20 lbs.
Households usually buying Oscar Mayer Wieners	39 /	42 /	35 /

Contd.→

	Total households	Listening households	Non-listening households
Oscar Mayer share of Wiener dollar among Oscar Mayer buyers	53 /	53 /	53 /
Total Oscar Mayer Wiener Consumption by these households (lbs.)	3,883,000	2,408,000	1,425,000
Less Oscar Mayer consumption not influenced by Braves advertising (pounds)	3,432,000	2,007,000	1,425,000
Increase due to advertising (lbs.)	401,000	401,000	0

SCHMALENSEE'S WORK ON THE ECONOMICS OF ADVERTISING

Schmalensee's[96] study is concerned with the impact of advertising on consumer behaviour and market performance. Its' unifying theme is that advertising expenditure must be treated as an endogenous variable in any investigation of its effects, and thus its determinants mustt be explicitly considered. After surveying the work of others and extending it in various directions, the study still reaches negative conclusions; there is barely a molehill of hard evidence behind the mountain of prose on the subject of advertising.

(Schmalensee's book is based on his doctoral dissertation "on the Economics of Advertising", which was presented to the Department of Economics, Massachusetts Institute of Technology, in January 1970. The dissertation was extensively revised and transformed into a book at the University of California, San Diego).

(1) Importance of Advertising - Sales Interaction Studies: Most business firms spend money to persuade potential costumers to buy their products. Advertising, while only one component of such selling costs, is an important part of the "marketing mix" for many firms. Also, advertising is a substantial part of the Gross National Product in developed countries. For these reasons, this study focusses on advertising to the exclusion of other marketing tools. According to "Printer's Ink", $ 16.6 billion were spent on advertising in the U.S. in 1966, about 2.2 / of current dollar G.N.P. "Advertising Age" placed aggregate U.S. advertising expenditures in 1966 at $ 16.8 billion.

(2) Normative and Positive Issues : Economists and others have levelled a variety of criticisms against advertising over the years. The main criticisms are : (i) the content and presentation of some advertisements is annoying or offensive. Positive Economics can contribute little to evaluations of these sorts of criticisms. (ii) Many economists have dealt with the welfare implications of advertising spending, as surveyed by Doyle (1968a). Begining with Kaldor (1950), economists considered the implications of the lack

of a separate market for advertising messages. Since advertising is supplied jointly with goods and services, and since consumers thus cannot directly indicate their willingness to pay for it, Kaldor argued that too much advertising is produced. Steiner (1966) agreed with this, stating that it is usually in the interest of producers to supply more advertising than consumers would demand even at a zero price. The cost of advertising is passed on to consumers through higher prices for advertised commodities, instead of having consumers pay for product information. Telser (1964, 1966) dissents from this reasoning. He feels existing economies of joint production would be lost with a separate market for advertising messages. [Since advertising is supplied jointly with goods and services, and since consumers thus cannot directly indicate their willingness to pay for it, Kaldor argued that too much advertising is produced]. In addition, Telser argues that, as long as there are unadvertised or lightly advertised goods available in the same market, the price differential between these and advertised goods cannot exceed the amount consumers are willing to pay for the extra advertising involved.

According to Schlemansee, "this contention seems basically correct". Even when one argues that consumers value advertising not as a source of information but for the taste - change it creates, it is clear there could be no over - advertising in a world of free entry, into all industries. Industries of this sort (with differentiated products, free entry, and a large number of firms) are usually termed, following Chamberlin (1933), monopolistically competitive.

(3) Alleged overadvertising by oligopolists : This argument effectively began with Marshall's (1922) distinction between "constructive" and "combative" advertising. While the former was described as conveying useful information, the latter was portrayed as socially wasteful competition for market shares. Many critics, such as Stuart Chase (1925) have charged that much advertising in industrialized economics is purely 'combative'. If the firms in some industry are advertising more than a profit-maximising monopolist would and industry sales are thereby affected, the firms would be better off if all would reduce advertising, but advertising is obviously affecting consumer's tastes, and a reduction in advertisement spending would induce a difficult-to-value alteration in preferences.

(4) The "Hidden Persuaders Thesis", after Vance Packard (1957). Packard's original charge has been amplified and extended in the writings of Galbraith (1958, 1967). These authors and others have agreed that because advertising influences the formation of consumer's tastes, it can influence spending decisions in major and undesirable ways. This argument requires hard, factual study.

(5) Advertising has been criticised as a source of monopoly power, generally through its impact on conditions of entry. Bain (1956) argues that the minimum efficient size of the firm is larger in some industries, due to economies of scale in advertising, and that advertising erects patterns of brand loyalty that work to the advantage of established firms. Comaner and Wilson (1967) argue, in

addition, that advertising tends to stabilise informal collusive price agreements.

Two important questions of fact have been raised here. First, are there economies of scale in advertising? This question in turn has two components : (1) whether successive advertisements in a given medium have increasing marginal effects on sales; and (2) whether the marginal cost of successive advertisements declines over any range. This second question hinges on the duration and stability of advertising's effects, on the half-life of investment in advertisings. If to-day's advertising does not affect tomorrow's sales, an entrant tomorrow need merely advertise at the same rate as existing firms to match their goodwill with consumers. On the other hand, if advertising leads to stable patterns of brand loyalty, it is as if firms were investing in slowly depreciating stocks of goodwill. An entrant must match the stocks of existing firms, and he can do this only by spending more than they do over a period of time. If they do not lower their advertising in the face of his entry, all firms may end, up advertising more relative to sales.

Schlemansee's study, while not exhaustive, has attempted to indicate what questions of fact must be answered before the main criticisms of advertising can be evaluated. Schlemansee's argument is that little of the required evidence currently exists. This is due in part to lack of effort by empirically - oriented economists, and in part due to data-limitations. But the main reason seems to

be that most studies do not adequately consider the determinants of advertising expenditure. In particular, discussions of firm's advertising decision rules reveal fundamental difficulties in measuring the effects of advertising on sales.

The measurement problem : "An individual firm's advertising seems likely to influence its sales, but it also seems quite certain that sales influence advertising for most firms". This stems from the common employment of a percentage-of-sales rule in advertising decisions. Given the mutual dependence of advertising and sales, simultaneous equations techniques must be employed in demand estimation. Failure to use such methods may produce biased and misleading estimates.

In spite of this, most of the relevant literature has used ordinary least square methods: e.g. Palda's (1964) oft-cited study. Palda's best-fitting estimate of the demand function for Lydia E. Pinkham Medicine Co 's products is the following estimated by ordinary least squares over the period 1908 - 1960. Sales = S_t; Advertising = A_t, T = time. Y = disposable personal income; D = a dummy variable which is 1 from 1908 until 1925 and 0 thereafter.

$$S_t = -3649 + 1180 \log_{10} A_t + 774 D + 32 T - 2.83 Y_t + .665 S_{t-1}$$

$$R^2 = .941 \quad DW = 1.59 = \text{Durbin-Watson Statistic.}$$

Melrose (1969) estimated several functions of this sort and

attempts to infer the parameters of Pinkham's demand curve from his least-squares estimates. However, if a function of this sort does describe how the company fixed its advertising budget, least squares estimates of demand functions or of advertising decision functions are inconsistent - least squares estimates are asymptotically biased.

Single-equation methods cannot generally yield reliable estimates of consumer response to advertising. To obtain such estimates, it is necessary to consider a simultaneous system, to pay attention to the firm's decision to advertise. And even then it may be impossible to identify a demand function.

Investigation and Findings. The unifying theme of Schmalensee's book is that any investigation of advertising cannot treat such spending as exogenous if misleading results are to be avoided. Rather, it must carefully take into account the manner in which firm's advertising outlays are determined.

Schmalensee critically surveyed previous studies of previous studies of advertising's effects on firm and industry sales, most of which fail to account for firm's advertising decision rules and none of which are wholly successful. The hypothesis about advertising's effects on sales which have been suggested are considered, but Schmalensee concluded that "no positive statements about the effects of advertising on demand can be made on the basis of the existing literature".

Schmalensee suggested that there is a need for more methodical work in the field of Advertising Effectiveness as resulting in sales.

Schmalensee studied advertising's impact on total cigarette sales in the United States and on the sales of each of the six major producers. A simultaneous equation estimator, designed to obtain consistent estimates of demand parameters, is presented and employed, along with single equation techniques. In spite of considerable effort, no truely reliable estimates are obtained, and the reasons for, and implications of, this failure are explored. "The results do suggest, though, that the impact of advertising on purchases of cigarettes, either in total or from individual firms, is less than popularly assumed".

Schmalensee applied the previous work to an important empirical question, the impact of advertising on the ability of firms and industries to earn monopoly profits. Schmalensee argued that a correlation between advertising intensity and profitability is to be expected even in the absence of a causal flow from advertising to profits. Further, it is shown that no evidence exists that advertising increases minimum efficient firm size or that advertising creates durable preference changes that serve as entry barriers to new firms. Schmalensee therefore argues that "nothing is really known about the impact of Advertising on monopoly profits".

SCHMALENSEE'S CRITICAL SURVEY OF STUDIES ON ADVERTISING'S EFFECT ON SALES

Palda (1964), Doyle (1968), and Montgomery and Urban (1969) have reviewed the existing literature, but none of them have subjected it to critical scrutiny, analysis and rigorous quantitative tests. Schmalensee has exhaustively criticised existing literature.

Schmalensee is of the opinion that most of the estimates of advertising's effects in existing literature were obtained using ordinary least squares. Such estimates are suspect and are unsatisfactory (e.g., Palda's work on Lydia Pinkham's Vegetable compound).

Simulation models of consumer behavior have been implemented, but this approach raises difficult questions about hypothesis testing and model verification. A major work in this area is that of Amstutz (1967). Montgomery and Urban (1968) and Kotler and Schultz (1970) have also worked in this area.

The literature on the Effect of Advertising on sales can be divided into two broad categories : those concerned with the absolute level of sales and those dealing with market share. The choice of approach depends on how seriously one takes Borden's (1942) distinction between "primary demand" (for total output of an Industry) and "selective demand" (demand experienced by particular firms).

The author of this thesis prefers the "selected demand" approach.

Simultaneous Equation bias : To the extent that current advertising depends upon current sales, least-square estimates of advertising's impact on demand will be inconsistent. In the simplest case, the nature and extent of the implied asymptotic bias can be directly evaluated. Let the true-demand and advertising decision equations be

$$S = a_1A + b_1X + c_1 + u_1$$

$$A = a_2S + b_2Z + c_2 + u_2$$

where A = Advertising,

S = Sales

X,Z are exogenous quantities distributed independently of the two error terms, u_1 and u_2.

a,b,c Constants which it is desired to estimate.

If the product of a_1 and a_2 is less than unity, the effect of increasing X will be to increase sales and advertising if b_1 is positive and to decrease both quantities if b_1 is negative. Similarly, the reduced form coefficients of Z will have the sign of b_2 if and only if this condition is satisfied. These seem sensible requirements. In terms of Fisher's (1970) "Correspondence Principle", the simultaneous system above can sensibly be considered as the limit of a non-simultaneous system as the time-lags involved approach zero if and only if $(a_1 \times a_2)$ is less than one. Schmalensee assumed that the product of a_1 and a_2 is less than unity.

Since advertising and the error-term in the demand equation are not distributed independently, ordinary least-squares will lead to inconsistent estimates of the parameters of the demand equations. Focussing on this relation, we have Bronfenbrenner's (1953) result for the asymptotic bias of the least squares estimate, a_1, of the parameter, a_1 : plim $(a_1 - a_1)$

$$= \frac{(1 - a_1 a_2)\ (\sigma_{12} + \sigma_{11} a_2)}{[\sigma_{22} + a_2^2 \sigma_{11} + 2 a_2 \sigma_{12}] + b_2^2 \,[M(z,z) - M(x,z)^2 \ M(x,x)]}$$

where σ_{ij} = covariance (u_i, u_j) $\qquad i, j = 1,2$

$$M(x,x) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x(t)^2 , \quad M(z,z) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} z(t)^2 \quad \text{and}$$

$$M(x,z) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x(t)\ z(t) \quad .$$

If a_2 is zero and if the two error-terms are uncorrelated; equations for S and A are a recursive system, and the least squares estimate of a_1 is consistent. In most situations, Schmalensee argues, a_2 is not zero, and the least squares estimates are asymptotically biased.

As long as the error covariance, σ_{12}, is small, then, the least squares estimates of a_1 is asymptotically biased upwards, implying that even if the true a_1 is zero, the probability limit

of the least-squares estimate of a_1 is positive. Any of the usual significance tests are useless in this situation, and least squares estimates will overstate the effects of advertising.

On the other hand, if the error covariance is negative and large enough, "we clearly may obtain a negative bias".

UTILITY THEORY AND PSYCHOPHYSICS

The usual discussions of consumer spending behavior assume that tastes are unchanged. The natural way to study advertising in this tradition is to investigate the consequences of changes in the parameters of the utility function. Basmann (1956) was the first to thoroughly explore this approach. By differentiating the first-order conditions for maximum utility with respect to a_j, where a_j is any parameter of the utility function, Basmann obtained

$$\frac{\partial x_i}{\partial a_j} = -\sum_{k=1}^{N} \frac{\partial MU(k)}{\partial a_j} \frac{S_{ik}}{\lambda} \quad ,$$

where x_i = the consumption of the i^{th} good,
$MU(k)$ = partial derivative of the utility function with respect to x_k, S_{ik} is the Slutsky-Hicks substitution term between goods i and k (the compensated cross-price effect), and $\lambda = \frac{MU(i)}{P_i}$ for all i. While only the S_{ik} terms onthe right of (1) are invariant against a monotonic transformation of the utility function, it is easily

shown that the summation on the right-hand side of (1) is invariant once one recalls that the sum over k of $P_k S_{ik}$ is always zero.

In an attempt to obtain more insight, Schmalensee considered a particular form of the utility functions, one in which advertising is 'consumption augmenting' in the following sense. He supposes that the utility function is of the form $[U[V'(a_1)x_1, \ldots\ldots\ldots V^n(a_n)x_n]$, where all the V^i are monotone increasing. (This parametrization of the utility function was first employed, though for a different purpose, by Fisher and Shell (1968)). In this model, an increase in the advertising of good j increases the apparent consumption of that good.

With U in this form, $MU(i) = U_i V^i$ and $\frac{\partial MU(i)}{\partial a_j} = V^i U_{ij} X_j V^{j'} + \delta_{ij} U_j V^{j'}$, where U_j indicates the partial derivative of U with respect to its j^{th} argument, $V^{j'}$ is the first derivative of V^j, and δ_{ij} is the Kronenker-delta function of i and j (i.e., δ_{ij} is one when i = j and 0 otherwise). Substituting these in the expressions for $\frac{\partial x_i}{\partial a_j}$, we obtain

$$\frac{\partial x_i}{\partial a_j} = - P_j \frac{V^{j'}}{V^j} \left[S_{ij} + \sum_{k=1}^{N} \frac{S_{ik} V^k U_{kj} x_j}{U_j} \right] .$$

This can be re-written in terms of the following elasticities

$$A_{ij} = \frac{\partial x_i}{\partial a_j}\frac{a_j}{x_i}\,; \qquad \sigma_{ij} = -\frac{S_{ij}P_j}{x_i}\,; \qquad n_j = \frac{V^j a_j}{V_j}\,;$$

$$\beta_{ij} = \frac{U_{ij}\,V^j x_j}{U_i}$$

The term A_{ij} is the (cross) Advertising elasticity of demand, n_j may be thought of as the augmentation elasticity of advertising on the jth good, σ_{ij} is the compensated (cross) price elasticity, and β_{ij} is the elasticity of the marginal utility of good i with respect to consumption of good j, all advertising expenditures constant. All these quantities except the β's are clearly invariant against monotonic transformations of U. Rewriting in terms of these elasticities, we obtain directly

$$A_{ij} = n_j \left[\quad {}_{ij} + \sum_{k=1}^{N} \sigma_{ik}\,\beta_{kj} \right]$$

If U is directly additive, all of the β's except for β_{ij} may be taken to be zero. It is then clear that the effect of consumption-augmenting advertising is almost a pure substitution effect. But even in the very special directly additive case, the presence of the β's makes it hard to see what sort of restrictions can be put on any estimates of A_{ij}, other than "adding-up" requirements of the usual sort, such as $\sum_i P_i X_i A_{ij} = 0$. And, if we are dealing with the demand function facing a firm or industry, additivity is an unreasonable assumption and the lack of a complete system robs the adding-up constraints of any value.

Massy (1960) and Haines (1969) point out an even more serious difficulty with the use of classical utility theory when tastes change - Let a_j be any parameter of the utility function, which may or may not be of the form discussed above. Then the partial derivative of any of the S_{ik} with respect to a_j will not be sign-invariant under monotone transformation of the utility function i. e. the direction of change of the substitution terms depends on the particular cardinal representation of the consumer's preferences employed. This result indicates that neo-classical consumer theory, derived on the basis of constant tastes, simply cannot be used as it stands in situations where tastes change.

A number of writers have attempted to deduce the form of consumer's response to advertising from the Weber-Fechner 'law' of psychophysics, (e.g. Benjamin and Maithand (1958) and Benjamin et al (1960). This proposition states that the perceived intensity of a stimulus, P, is related to the actual intensity of the stimulus A_1 according to $P = a \ln(ba)$ over some relevant range (Stevens, 1968). Even if the law holds in psychophysics, extending it to advertising - sales interaction requires tremendously bold assumptions. "Perceived advertising intensity" may bear no relation to purchasing patterns; it is certainly unlikely that this relation is simple. Besides, it might be argued that price and income are also stimuli, on a par with advertising. And we know from a number of studies that the relation between actual prices, perceived value, and purchasing behavior is quite complex - Important studies are those by Tull (1964), Gabor and Granger (1966), Stafford and

Enis (1969). "Thus neither utility theory nor psycholophysics can tell us much about the sort of models to employ to detect the relationship between advertising and sales.

MARKET SHARE MODELS

The fact that market shares, like probabilities of mutually exclusive and collectively - exhaustive events, must add to one has induced a number of authors to apply stochastic process models to the study of brand choice (listed by Montgomery and Urban, 1969, and Jones, 1970). In order of their introduction, the most common of these are zero-order Bernoulli, first-order Markov, and linear learning models.

The Zero-order Bernoulli Model :

The zero-order Bernoulli Model states that the probability that the k^{th} consumer will purchase the i^{th} brand, p_i^k, is constant over time in the absence of changes in price or advertising inthe relevant market. The expected-market share of each brand is thus also constant.

The first-order Markov Model :

The first-order Markov Model assumes that the probability that any household will purchase any one of the brands in the market considered is a function of the last brand bought. That is, the probability that the k^{th} consumer will switch from brand i to brand j is given by p_{ik}^j. The matrix of these probabilities is

called a transition matrix, and it is assumed constant when external influences (price, distribution, advertising, etc.) are unchanged. In most applications, a common transition matrix is assumed for all consumers.

The linear learning model :

The linear learning model is based on work in psychology (Bush and Mosteller 1955). The hypothesis is that for the k^{th} household the probability of purchasing brand i in period $(t+1)$ is given by $p_i^k(t+1) = a_i^k + b_i^k p_i^k(t)$ if brand i was purchased in period t, and by $p_i^k(t+1) = c_i^k - d_i^k p_i^k(t)$ if brand i was not purchased in period t, where a_i^k, b_i^k, c_i^k and d_i^k are positive constants. These two equations are referred to in the literature as gain and loss operators. As above, this model assumes no change in prices or advertising outlays.

Styan and Smith (1964) and Massy (1966) found the Markov model worked better than the Bernoulli approach for aggregate data. On the other hand, Massy found that the hypothesis that individual families had constant purchase probabilities (the Bernoulli model) could not be rejected for most families in his sample. For those families for which it was clear that the Markov model was appropriate, the hypothesis of identical transition matrices could be rejected. These and other results have led several authors to warn against simplistic application of the Markov model. Howard (1963) investigates the problems of aggregation and interpurchase timing in this

model. Mercer (1966) and Ehrenberg have been critical of the Markov approach, Ehrenberg (1965), Massy and Morrison (1968), and Ehrenberg (1968).

Kuehn (1948, 1962) and Carman (1966) have found that the linear learning model provides good fits to individual family data. However, Frank (1962) has shown that such results can emerge from a Bernoulli process with different purchase probabilities among households (Montgomery and Urban, 1969). This suggests that non-Bernoulli processes may have serious problems on the individual level, but that for one reason or another, they explain aggregate data fairly well. Most authors have taken such models as a starting point when attempting to explain market shares when externalconditions vary over time.

Kuehn (1969) has suggested a solution to the problem of aggregating the linear learning model. (Haines, 1964 has also suggested a solution, but it is applicable only to new products). Under Kuehn's assumption, this model is indistinguishable from a first-order Markov model.

Kuehn assumes that the transition matrix in a Markov model is the same for all families and that

$$P_{ij} = \delta_{ij} r + (1 - r)a_j \quad ,$$

where δ_{ij} is the Kroneker delta function of i and j. The parameter r may be thought of as the fraction of consumers who will never

switch brands. The fraction a_{ij} is the percentage of switchers who move to j^{th} brand; the sum of the a_{ij} must obviously be unity. By definition

$$P_i(t+1) = \sum_j {}' \, p_{ji} \, P_j(t)$$

where the summation is over all brands in the market. Since $P_i(t)$ is the probability that each consumer will buy brand i, we can identify it with that brand's market share, $MS_i(t)$. Making this substitution, $MS_i(t+1) = r\,MS_i(t) + (1-r)a_i$, with steady-state market shares being the a_i's.

Kuehn writes the gain and loss operators for a linear learning model for the i^{th} brand in the following form, where g_i and k_i are positive constants :

$$P_i(t+1) = P_i(t) + g_i\left[\bar{U}_i - P_i(t)\right] = Y_i\left[\bar{P}_i(t)\right] \quad \text{(gain)}$$

$$P_i(t+1) = P_i(t) - k_i\left[\bar{P}_i(t) - L_i\right] = N_i\left[\bar{P}_i(t)\right] \quad \text{(loss)}$$

The first of these gives the change in P_i when the i^{th} brand is purchased in period t. The second equation applies when any other brand is bought in that period. It should be clear that U_i is the limit approached by P_i as the i^{th} brand is purchased without interruption for a large number of periods. Similarly, L_i is the lower limit of P_i as brand i is not purchased for a

large number of periods. If the number of brands in the market is constant, we have $1 - U_i = \sum_{j \neq i} L_j$, for all i.

Kuehn assumes that g_i and K_i do not vary across brands i.e. he assumes identical learning mechanisms for all brands. He cites empirical evidence for this in the case of established brands, and then asserts that in order for the sum of the P_i's to always equal one, we must have $g = k$.

Banks (1961) sought to explain the market shares of nine brands of cleanser and twenty-one brands of Coffee in Chicago in December of 1950. He employed seven independent variables, none of which were considered to sum to a fixed amount across brands at all times (sum constrained). The coefficients were the same for all brands. For cleansers, the coefficient of dollar media advertising expenditures was significantly positive, but point of purchase advertising had a significant negative effect. For coffee, point of purchase advertising had no significant impact, and the coefficient of media advertising was negative and significant.

In a study of cigarette advertising, Telser (1962) estimated the following equation separately for Camels, Lucky strike and Chesterfield, using data for 1929-1939 :

$$MS_i(t) = a_i + b_i S_i(t) + c_i MS_i(t-1) ,$$

where $S_i(t)$ was advertising on brand i divided by advertising on all other brands. Clearly this variable was not sum - constrained.

There was considerable variation in the coefficients across brands. For the period 1954-59, a similar equation was estimated for each of the six major cigarette companies, with the addition of a "Product-mix-Index" term. The coefficients of $S_i(t)$ were significantly greater than zero for only two of the six firms.

Buzzell (1964) attempted to measure the effects of varying advertising quality as well as dollar expenditures. He found that advertisement quality, as measured by the reactions of randomly - selected audiences, was more important than advertising share in explaining market share. In some regressions, the coefficients of the latter were insignificant. Because the testing system Buzzell employed to measure advertisement quality was not universally accepted, and for other reasons, these findings have been subjected to considerable criticism. Fothergill and Ehrenberg (1957), Buzzell (1965) and Murphy and Buzzell (1965) have done significant work in this area.

Weiss (1968) attempted to measure the influence of price and advertising on the market shares of three brands of a low-cost consumer food item. He estimated variants of the following model :

$$MS_i(t) = a_i + bP_i(t) + cA_i(t)$$

The lack of a lagged dependent variable is puzzling in view of earlier work. The variables P_i and A_i measured price and advertising, either absolute or relative. In no version of the model were these variables sum-constrained. When the a_i's were allowed to be

different for different brands, none of the advertising variables employed were significant.

Peles (1971) estimated a number of equations of the following form for the auto, beer and cigarette industries : $MS_i(t) = a + bS_i(t) + cMS_i(t-1)$, where $S_i(t)$ is the i^{th} firm's or the i^{th} brands share of total industry advertising. Experiments were also performed with relative price terms and lagged values of $S_i(t)$. In all of these equations, the coefficient of $S_i(t)$ was significant.

Finally, Simon (1969) examines sales of fifteen popular liquor brands in the period 1953-62. His model is

$$MS_i(t) = a_i MS_i(t-1) + b_i \log\left[A_i(t)\right],$$

where $MS_i(t)$ is sales of the i^{th} brand divided by total sales of brands of its type (all gins, for instance), and $A_i(t)$ is dollar advertising spending on the i^{th} brand in year t. The advertising variable is obviously not sum-constrained. The t-statistic corresponding to this term exceeded two for only five brands.

All of these studies neglect the simultaneous equations problem. In addition, none constrain the sum of predicted market shares to equal unity. The models employed can be summarised by the following equation

$$MS_i(t) = a_i + \sum_{j=1}^{k} b_i^j \times i^j(t) \quad ,$$

where the k independent variables may include lagged market share. In all the studies cited here : (1) b_k^j was not constrained to equal b_m^j for $k \neq m$, and/or (2) one or more of the x^j was not constrained to sum to a constant value over all products in the market, and/or (3) no relation between the a_i and the b_i^j was imposed. Schmalensee showed that the presence of any of these three conditions implies that the market shares predicted from the estimated equations will not in general sum to unity across all products. Since this constraint is not imposed, serious doubt is cast upon the meaning of the results presented in all of the studies cited above. They are at best historical summaries, since none fulfills conditions that must hold for the true structure if a linear market share model is appropriate.

Suppose that the sum of the market shares predicted by the equation $MS_i(t) = a_i + \sum_{j=1}^{m} b_i^j \times i^j(t)$ is unity for all admissible data. Summing across the N firms or brands then yields $1 = \sum_{i=1}^{N} a_i + \sum_{j=1}^{K} b^{j'} x^j$. If for some j there are N linearly independent admissible x^j such that $u' x^j = s$, and if the sum of the market shares predicted is unity for all admissible x^j, one and only one of the following is true: (a) $b^j = 0$ (b) $b^j \neq 0$ and x^j is sum-constrained. Again, suppose for each j there are N linearly independent x^j such that $u' x^j = s^{j'}$. Then the sum of the market shares predicted will equal unity for all admissible

data, if and only if

(a) x^j is sum - constrained, for $j = 1, \ldots\ldots, k$.

(b) $b^j = \beta^j u$, β^j a scalar, for $j = 1, \ldots\ldots, k$.

(c) $$\sum_{i=1}^{N} a_i = 1 - \sum_{j=1}^{K} \beta^j s^j$$

Sexton (1970) investigates a number of non-linear market share models that also fail to constrain the sum of the predicted market shares to equal unity. His preferred equation is

$$\log\left[1 - MS_i(t)\right] = a_i + b_i \log\left[1 - MS_i(t-1)\right] + \sum_{j=1}^{J} c_{ij} A_i^j(t) + \sum_{k=1}^{k} d_{ik} Z_i^k(t) .$$

Here MS_i is the i^{th} brand's (or products) share of total product (or product class) sales. A_i^j is advertising of the i^{th} brand (or product) in the j^{th} medium, and Z_i is a variable designed to capture the effects of the i^{th} brand's (or product's) average price or average 'deal' price. The a priori necessary constraint is that the sum of one minus the anti-lags of the dependent variables be one.

Sexton (1970) presents estimates of this model using consumer panel data on sales of a brand, the corresponding product, and the corresponding products class. Using seventy-nine weeks of data,

none of the c_{ij} are significantly different from zero. He estimates a number of other brand share and product share models, both linear and log-linear and all unconstrained, using the same data. Significant coefficients of current advertising variables have the wrong sign more often than not, and most advertising coefficients are not significantly different from zero.

Only two published studies of market share behavior satisfy the conditions mentioned in the preceeding paragraphs. Both are applications of the following general model proposed by Kuehn and Weiss (1965).

$$MS_i(t) = rMS_i(t-1) + (1-r)\left[b\,\frac{K_i[P_i(t)]^{-e_p}}{\sum_j K_j[P_j(t)]^{-e_p}} + (1-b)\,\frac{K_i[P_i(t)]^{-e_p}[S_i(t)]^{e_a}}{\sum_j K_j[P_j(t)]^{-e_p}[S_j(t)]^{e_a}} \right]$$

The constant r reflects the force of inertia on purchase decisions, and b is the fraction of the market which is not influenced by advertising. The parameters e_p and e_a measure the sensitivity of demand to price and advertising, while the K_i allow the 'intrinsic appeal' of the brands to differ. The variable $S_i(t)$ is either the i^{th} firms (or the i^{th} brands) advertising in period t or its stock of goodwill, calculated by depreciating past advertising outlays.

In the first study using this model, Kuehn (1969) set the parameter r equal to zero, thus assuming no habit formation or inertia. The model was fitted to four years of bimonthly advertising, price and market share data for four major brands of a grocery product in one metropolitan market. Parameters were chosen using a search technique to minimise the sum of squared residuals. No tests of significance were presented. The estimated value of b was 0.925, indicating that most purchases were not affected by advertising. The long-run advertising elasticities of market share for all four brands were estimated to be less than 0.18. Thus, even when no habit formation was assumed, the influence of advertising was estimated to be "slight".

Weiss (1969) performed a similar analysis of bimonthly data for three brands of a low cost food item in the Chicago metropolitan area. He, too, assumed $r = 0$. His estimate of b was 0.957, implying even less influence for advertising. In fact, his estimates of the advertising elasticity of market share were all well below 0.01. As above, no significance tests were presented. Weiss did mention a set of experiments using lagged advertising in place of current advertising but the results were not sensible.

STUDIES OF FIRM AND INDUSTRY SALES

Schoenberg (1933) estimated the following demand function for cigarettes using data for 1923-31

$$X = 1258.0 - 80.4Y + 7.9W + 47.1t$$

Standard errors	(10.7)	(4.4)	(10.8)
t - statistics	-7.5	1.8	4.36

$R^2 = .996$

Standard error of the regression SEE = 12.0

X = cigarette consumption per capita

Y = wholesale price of camels per thousand deflated by the wholesale price index

W = total newspaper advertising by the four leading cigarette manufacturers

t = time measured in years.

The coefficient of W is significant at the 10 / level.

Meissner (1961) reported on a Stanford Research Institute investigation into the sales of lettuce. A multiple regression analysis was carried out using annual data from 1950 - 55 on twenty-two cities. All observations were pooled, and the dependent variable was per capita consumption of lettuce. There were three non-advertising independent variables: income, price and average maximum temperature. Seven additional independent variables were used to measure different phases of the overall marketing effort carried out by Lettuce, Inc., an association of lettuce shippers in Salinas, California. No lagged variables were employed. At the 10 / level, price and temperature had significant negative effects, which only one of the marketing variables, an index of newspaper cooperation, had significant positive

impact. 63 / of the variance of the dependent variable was explained. The joint effect of the seven marketing variables was significant at the 5 / level, but the imprecision of the coefficient estimates made it impossible to say more than this.

Nerlove and Waugh (1961) carried out a very interesting and careful study of the advertising of oranges. They presented the following estimate of the demand function, fitted to crop years 1907-08 through 1940-41 and 1946-47 through 1958-59

$$V_t = -2.929 - \underset{\substack{(.198)\\-1.97}}{.390}\, Q_t + \underset{\substack{(.191)\\4.84}}{.924}\, Y_t + \underset{\substack{(.125)\\1.86}}{.233}\, A_t + \underset{\substack{(.045)\\2.29}}{.103}\, \bar{A}_t$$

$R^2 = .72.$

The variable V_t is the logarithm of the per-capita farm value of orange sales deflated by the consumer price index in year t. The independent variables Q_t, Y_t and A_t are logarithms of the following quantities: per-capita disposable income of consumers deflated by the consumer price index in year t, and per capita advertising expenditures by Sunkist Growers and the Florida Citrus Commission (producer Cooperatives which account for the bulk of the marketings) deflated by the consumer price index. The last variable, $\bar{A}_t$, is the logarithm of the average of deflated per capita advertising expenditures for the ten years preceding year t. All coefficients of the independent variables are significant at the 5 / level.

Only advertising has dynamic effects on demand. The authors state that they attempted to use a Koyck lag, but that the results were totally unsatisfactory. The short-run advertising elasticity of demand is 0.17 and the long-run elasticity 0.24.

Nerlove and Waugh fit the equation $V_t = -2.929 - .390Q_t + .924Y_t + .233A_t + .103\bar{A}_t$ to the subperiods 1909-36 and 1937-58. In the first of these only the coefficients of Y_t and $\bar{A}_t$ are significant, while inthe second subperiod only Y_t has a significant coefficient. Striking differences in the coefficients of the two regressions call into question the stability of the relationship, and suggest that analysis of covariance should be used to test for possibility of structural change.

Taylor (1968) estimated the coefficients of the following models for twenty-two categories of consumption using annual data for 1946-64.

$$Q_t = a + b_1Y_{t-1} + b_2A_{t-1} + b_3P_{t-1}$$

$$Q_t = a + b_1Q_{t-1} + b_2\Delta A_t + b_3A_{t-1} + b_4\Delta Y_t + b_5Y_{t-1}$$

$$Q_t = a + b_1Q_{t-1} + b_2Y_{6-2} + b_3A_1$$

$$Q_t = a + b_1Y_{t-1} + b_2\bar{A}_t + b_3P_t$$

$$\ln Q_t = a + b_1 \ln Y_{t-1} + b_2 \ln\bar{A}_t + b_3 \ln P_t .$$

The dependent variable, Q_t, was per capita consumption in the category in question in 1958 dollars. The income variable,

Y_t, was taken as total per capital consumption in 1958 dollars, and P_t was either the implicit deflator for the category in question or the category's deflator divided by the implicit deflator for total consumption.

The Advertising variable, A_t, was then calculated by deflating the estimated dollar outlays for each product class by the implicit deflator for gross national product. The variable $\bar{A}_t$ appearing in models was a 3 - year moving average of A_t. Advertising, curiously, was not expressed in per-capita terms, even though consumption and income were.

Peles (1971) examined the effects of advertising on the total sales of the beer, cigarette and automobile industries. The impact of advertising was estimated to be of significant import only in the case of automobiles, but the results were suspect since Peles did not consider the stock of automobiles on road.

Vaile (1927) observed that during the 1920-22 recession, firms which cut their magazine advertising showed greater percentage sales decline than firms which never appeared in magazines. Those which never advertised, in turn, suffered loss in percentage terms than firms which cut their advertising. This pattern held within as well as across industries. No tests of significance were reported.

Cowan (1936), observing that in 1927 most Chevrolet advertising was done in the Saturday Evening Post, examined the relationship between Chevrolet registrations per capita and Post circulation

per capita across a large number of countries. He found a strong positive association, but nothing in his analysis refuted the obvious alternative hypothesis that both variables depended strongly on per capita income and that there was no causal relation between them.

Cowan also examined a product whose sales were declining in spite of a network radio advertising campaign. He found a negative association between the percentage decline in sales and the number of radios per capita across countries. This result was a bit less suspect than the one above, but no evidence was presented that radio ownership and disenchantment with the product were not both simply functions of income.

In a study of department store advertising, Brown and Mancina (1940) found "no definite tendency for maximum sales to occur at any ratio of advertising expense to sales".

Berreman (1943) employed correlation analysis to examine the impact of pre- and post-publication advertising on the sale of novels in the 1930's. The sales of novels which did not make it into the 'best-seller' category were quite strongly related to the amount of pre-publication advertising. Most best-seller received more pre-publishing advertising than novels which did not sell well, but within the class of best sellers there was no relation between pre-publication advertising and later sales. Within the class of best sellers, there was a significant relationship between post-

publication advertising and sales, but Berreman stressed that no causal conclusions could be drawn from this correlation. This was one of the first statements of the simultaneous equations problem to appear in the advertising literature, albeit implicitly.

Roberts (1947) studied the impact of advertising on the sales of two drug products, which he denoted as A and B. Roberts attempted to measure the cross-effects of advertising as well as direct impact, i.e., B's advertising was included in the equation explaining A's sales, and vice-versa. The other unique thing about this work was that Roberts had detailed information available on 1,504 families for six-months periods. The dependent variables in his regressions were thus the total purchases of A and B by each family during a six-month period. Non-advertising independent variables were city size, age of head of household, number of people in the family, economic class, education, occupation and region. All except number of people were coded into groups. Both A and B advertised only in magazines, and the size of the advertisements did not vary much, so a logical measure of advertising on the family level was the number of magazines carrying advertisements for A or B that entered the home during the six-month period. Unfortunately, advertising of A was highly correlated ($r = .84$) with advertising of B. The regression explaining A's sales had an R^2 of 0.045, not surprisingly low for data of this type. The coefficients of A's advertising was positive and significant at the 1 / level. The coefficients of B's advertising was negative, as expected, but not significant at the 10 / level.

The regression with B's sales as the dependent variable had an R^2 of only 0.035, and neither advertising variable was significant.

Benjamin and Maitland (1958) analysed four different models of sales response to advertising, using data from five different advertising campaigns. The best equation was of the form $Q = a + b \ln A$ where Q is sales response in units of products, and A is advertising expenditure. Paldas' work (1964) has already been discussed. Delymple (1968) reported an analysis of the sales of clothing items in department stores. However, advertising did not enter significantly into regressions that explained sales.

Lambin (1969) examined the demand for a frequently purchased food product sold in Belgium. It was a dominant firm, so Lambin neglected small competitor's activities. Over a twenty-year period an equation involving disposable income, lagged sales and advertising all in per capita terms, and rainfall and distribution explained 97.4 / of the variance in per capita sales. The logarithm of per capita advertising was used. All coefficients were significant and had the expected signs.

Peles (1971) estimated demand equations for firms and brands in the beer, cigarette and automobile industries. The estimates for automobiles were marred by a failure to adequately deal with the effects of the stock of cars on the road, and the results were generally not sensible.

Bass (1969), using a simultaneous equations study of advertising's effect on demand, examined the per capita sales of regular and

filter cigarettes. He related sales in each category to advertising in each category, per capita disposable income, and the price of non-filter cigarettes divided by the consumer price index.

Bass and Parsons (1969) studied a class of frequently purchased products sold predominantly in supermarkets. Again, Bass and Parsons, using a four - equation, log-linear model, studied the same class of products. Bass and Parsons, in their simultaneous equation study found that all coefficients of advertising are positive. In fact, if the estimates are to be believed, doubling the advertising of the remainder would increase sales of brand SL-6 by 48.2 / , whereas doubling the advertising of SL-6 would add only 22.1 / to its sales. (The model was expressed in terms of one brand of interest, RH-2, and the Remainder defined as all other established brands).

DISTRIBUTED LAG MECHANISMS

(Effect of Advertising persisting over time)

Jastram (1955), discussing this concept, cites references to it written as long ago as 1931. Tull (1965) summarised "the evidence ..... while tending to support the belief that such an effect exists, is persuasive but not conclusive".

Vidale and Wolfe (1957), and Benjamin et al. (1960) argued for a more complex lag structure.

Benjamin proposed the following model for sales response to an isolated advertisement.

$Q(t) = a\, e^{-bt}(1 + c\,t)^{d}$. t = time since advertisement, a, b, c, d are constants.

Ozga (1960) and Stigler (1961) has proposed models for sales response to advertising based on diffusion and contagion. These processes have been analyzed by Gould (1970).

Ozga's formulation is

$$\frac{d\,Q(t)}{dt} = \frac{b\,[K - Q(t) - K\,Q(t) + Q(t)\,Q^{*}]}{(K - Q^{*})}$$

K = Potential sales. Q^{*} = equilibrium sales, t = time, b = constant. The path to equilibrium is a S-shaped logistic curve, with the most rapid increase in Q occuring at $t = t = c_2/\ln c_1$. When t' is near 0, we have approximately exponential paths between steady states.

The Stigler model is based on contagion phenomena

$$\frac{d\,Q(t)}{dt} = \frac{b\,[K(Q^{*} - Q(t) + 2Q^{*}\,Q(t)]}{(K - Q^{*})}.$$

The solution to the Stigler model yields an exponential approach to $\dot{Q}$, just as the Koyck formulation.

All of this work suggests that the Koyck or Geometric lag structure may be a fair approximation to the dynamic impact of advertising when the time periods involved are fairly long.

A STUDY ON TV ADVERTISING AND SALES

In 1959, Schwerin Corporation began a systematic investigation of relationships between TV advertising and sales. Kudisch[97] reported this study in 1965.

"Televison campaign effectiveness" data are obtained in the following manner. An audience of 300 or more is attracted to the test centre by random mailing, the procedure being basically the same in all the Schwerin locations in the United States, Canada, United Kingdom, and Western Europe. A sample weighted on demographic and brand preference factors is subsequently drawn from this random audience.

After filling out a questionaire, the respondents select from a list of brands in the particular product field the one they wish to receive if they are the winners in a drawing; a substantial supply, e.g. $ 10 worth of detergent, is offered to encourage a considered decision.

The audience then views a commercial for one of the brands in a 30 - minute control program, along with two other commercials for unrelated products. Following this, they again make a selection from the same list. The net percentage of the sample changing their choice of the advertised brand is the effectiveness score or preference charge.

As a further step, the score for each commercial (or a group of commercials representing a campaign) is compared with the current

average for the product field to determine whether it is superior, equal, or inferior to the competition.

The Preliminary Investigation : In the original phase of the study, changes in Television advertising expenditures were compared with changes in sales, using four-month or six-month periods, depending on how the information was customarily reported in the product field. A χ-square analysis was then applied; it revealed that the probability of any relationship between the amount spent and sales was at the 50 / confidence level.

The same comparison was also made between Schwerin preference-change scores and sales; and this time the Chi-square analysis yielded a highly positive result, at better than the 99 / confidence level.

Product fields included in the study were confined to consumer packaged goods that used Television as their dominant advertising medium. All periods for which measurements were available in each of these product fields were included. Data on a total of 553 "sales - advertising" periods were assembled. In addition to the χ-square values, the results showed that :

(i) A majority of the "sales-advertising" periods that had been rated as superior in terms of Schwerin preference-change scores actually experienced sales increases, while a majority of those rated "inferior" registered sales declines.

(ii) Expenditure increases and decreases apparently did not

have a similar distinct association with sales changes, although there was some tendency for periods with "superior" commercials to show particularly large sales gains when supported by increased expenditures.

(iii) There were some "maverick" cases which did not conform to the typical, observed pattern. "Further investigation of these situations led to the development of the "momentum" variable, which was incorporated in our later multiple regression model"[98]

The concept of "momentum", as here employed, has been described elsewhere[99]. Briefly, it is based on the hypotheses that consumer's attitudes toward individual brands lead their purchasing of these brands.

Computationally, it is a brand's attitudinal share (as measured by the Schwerin pre-choice percentage) minus its market - share at a given time. Thus "momentum" represents the extent to which a brands' sales share is lagging behind its share of favourable consumer attitudes. "Momentum" has a positive effect - in terms of implying future market share gains - when a brands share of favourable predispositions is greater than its sales share.

The Advanced Study :

The second study took the form of a 4-variable multiple regression model. Period-to-period sales share change was defined as the dependent variable, and the three independent variables were : TV advertising, expenditure share - change, SRC preference change, and "momentum".

Another product field was added to the original study, bringing the total measurement periods to 67. Throughout the seven product-fields analyzed, an average of 73 / of the sales - share movement was explained by the three independent variables acting in concert. Expenditure-share change contributed an average of 8 / of this explanation; Television commercial quality contributed 31 / ; and "momentum" 34 / (see Schwerin study[100]).

Net regression coefficients were derived for each independent variable in every product field, and for the seven fields together, so that forecasts of sales - share changes under varying conditions might be made. (use of regression analysis enabled the attribution of a degree of explanation to expenditure that was not possible with the χ-square type of analysis used in the preliminary investigation.

Applying the model's findings : with positive or negative values for the three variables, there are eight possible combinations. These range from momentums, campaign quality, and spending - all working in the advertiser's favour - to the nagative opposite. The average effect on sales of the various possibilities is shown in Table (i).

Table (i) represents the results of a 67-case model - averages from all the product fields studied.

Table (i)

Average Effect of three variables on sales- share change

Momentum	Quality	Spending	Mean Expenditure share-change	Mean sales - share-change
	+	+	+ 6.58	+ 0.71
+		-	- 2.68	+ 0.24
	-	+	+ 3.86	+ 0.06
		-	- 2.04	-0.17
	+	+	+ 3.20	+ 0.10
-		-	- 3.38	- 0.26
	-	+	+ 3.38	- 0.62
		-	- 2.86	- 0.78

Another example is from the Beer-product field. About $ 2.25 billion of beer was sold in the U.S. during 1963, and one well-known brand had approximately a 9 / share of the Industry total.

Table (ii) shows the theoretical extremes, and the gradations between them, for a brand with this share of market in terms of sales in this particular product field.

Table (ii)

Expected Sales Movements - Beer Product Field

Momentum	Quality	Spending	Expected change	Sales movements Annual $ gains/losses
	+ (+)	+	+ 7.9	+ $ 15,975,000
+	+	-	- 2.7	+ 5,400,000
	- (-)	+	+ 0.7	+ 1,350,000
	-	-	- 1.9	- 3,825,000
	+	+	+ 1.1	+ 2,250,000
-		-	-2.9	- 5,850,000
	-	+	-6.9	- 13,950,000
	-	-	-8.7	- 17,550,000

THE WOODSIDE - WADDLE STUDY ON SALES EFFECTS OF IN - STORE ADVERTISING (1975)

Woodside and Waddle[101] found that point-of-sale promotion more than doubled the effect of price-reduction on until sales of instant coffee. They began by asking the following questions, will the use of in-store advertising at regular prices produce greater sales than the use of price specials without advertising? How

significant are the synergistic effects on product sales if both a price special and in-store advertising are used?

After reviewing several empirical studies, Engel, Kollat, and Blackwell[102] point out that consumers find it difficult to know prices when confronted with thousands of items. Sinse most items tested are purchased frequently in supermarkets, it seems logical to assume that consumers have even less knowledge of actual prices of the vast majority of products. If a store wants to create a low price impression, regular price along with special price should be used in advertising and point-of-purchase displays.

A number of studies appearing in "Progressive Grocer"[103], 1971 resulted in point-of-purchase displays increasing sales from 18 to 400 per cent more than normal shelf movement.

A study in "Grocery Manufacturer" (1971)[104] utilised controlled merchandising tests in a panel of A and P supermarkets to demonstrate the importance of different point-of-sale displays for new products. Their tests indicate that use of a simple sign has decreased as a point-of-sale method since discounting has become more general but that it is still effective in increasing sales. The specific findings included :

Using hand-lettered cards, which designated the shelved item "As Advertised" moved from 61 per cent to 177 per cent more product than in control stores which did not use shelf signs. In another test of shelf signs, results indicated that their selling effectiveness

seems to depend on how well the sign informs the customer. Signs reading "save more" were put on shelves in front of certain items, but sales varied only from no change to 9 per cent higher. When the name and type of product were added to the sign (without price) sales rose from 5 per cent to 31 per cent. However, when container size and selling price were added, movement climbed 10 per cent to 59 per cent.

Two complementary rationales have been suggested to explain the efficacy of in-store advertising in increasing sales (Engel, Kollat, and Blackwell, 1973).

The exposure hypothesis maintains that end-aisle displays and for point-of-purchase material increase the percentage of customers who are exposed to the products in question.

The comparison-process hypothesis maintains that a substantial proportion of customers think that prices have been reduced on display items. For many consumers, buying items on display is a response trait or a performance strategy that is used indirectly to evaluate alternatives.

Woodside and Waddle (1975) examined three hypotheses:

Hypothesis I : The presence of point-of-sale advertising is more effective in increasing the sales of a product than is a price reduction.

Hypothesis II : Consumers will buy more of a product at a reduced price than at the normal price with point-of-sale

advertising present or absent in both instances.

Hypothesis III : Consumers will buy more of a product at the normal price when point-of-sale advertising is present than they will buy at the normal price when no point-of-sale advertising is present.

Method :

A Latin square design was used, thereby controlling simultaneously the effects of two independent variables, such as store size and time period.

Four supermarkets and four consecutive weeks in a period free of holidays were used in this study. A fifth week was added to the design to measure carry-over effects of the treatments.

A price reduction of approximately 20 per cent was chosen for this experiment. This price special was assumed to be a significant reduction from the point of view of both the retailer and the consumer.

The type of point-of-sale advertising utilised was a simple, hand-lettered sign attached to the normal shelf space of the product. This was a relatively crude form of point of-sale advertising as compared to other forms which are found in retail stores. The sign was chosen because it was felt to be the least effective type which could possibly be utilised. This choice was to make the hypothesised effectiveness of point-of-sale advertising difficult to attain. If significant effectiveness was found while utilising only this minimum of the potential power which point-of-

sale advertising holds, a robust test would substantiate the first hypothesis.

The sign was a 5 by 7 inch hand-lettered card showing the brand name, size, price, "No limit", and a 1 by 2 inch sticker of the package.

Product :

The product involved in the experiment was instant coffee. This is a product for which previous research (Woodside, 1973; Lambert, 1970; Gabor and Granger, 1964) leads to the expectation of exceptionally good sales response to a price special. Consumers have been found to show awareness of and concern for coffee prices and not to judge coffee quality by price. Coffee was chosen in order to make the expected relative ineffectiveness of the price reduction variable particularly difficult to achieve.

Four supermarkets in the same chain organisation were used in the study. The managers agreed to participate and allow daily audits of their coffee sales. The store managers were reimbursed for the difference of the price special from the regular price for units sold in the price special treatments.

This experimental study is constructed to offer information on the relative effects of price specials and point-of-sale advertising on unit sales of consumer products. It is limited to one type of point-of-sale advertising, one consumer product, and the use of four stores in one city.

Results :

The results of the experiment with the extra period included are presented in Table (i). The data are units of the product sold.

Table (i) : Results of Extra-Period Latin-Square Design Experiment in Product Unit Sales

Time Period \ Retail Store	A		B		C		D	
I	T_3	30	T_1	14	T_2	22	T_4	51
II	T_4	70	T_2	27	T_3	34	T_1	11
III	T_2	23	T_4	84	T_1	19	T_3	32
IV	T_1	19	T_3	31	T_4	85	T_2	23
V	T_1	25	T_3	36	T_4	45*	T_2	23

Overall Total Unit Sales (o) = 704

Overall Mean Unit Sales (o) = 35 - 20

In Table (i), T_1 = No price reduction, no promotion; T_2 = price reduction, no promotion; T_3 = No price reduction, promotion; T_4 = Price reduction, promotion. Again, * denotes a stockout of the item which occurred in store C during the fifth week.

Results of the analysis of variance for the data obtained in the extra-period Latin-Square design are shown in Table 2(ii).

Table 2(ii) : Results of Analysis of Variance of Extra-Period Latin-Square Design Experiment

Source of Variation	Sum of Squares	Degrees Freedom	Mean Square	F Ratio	Probability > F
Time Period	324.696	4	81.174	.886	.5253
Retail Store	494.80	3	164.933	1.801	.2471
Direct Treatment	7030.91	3	2343.636	25.596	.0008
Carry Over	427.42	3	142.473	1.556	.2947
Residual Error	549.374	6	91.562		
Total	8827.20	19			

The treatment carry-over detected in the Lain-Square with the extra period was not significant. Unit sales of the product were affected by the current treatment significantly with no significant effect being brought about by any treatment in the preceding time period.

Further investigation of carry-over effect was warranted to determine whether the carry-over effect from any one treatment may significantly differ from any other. The presence of any significant carry-over effect would attenuate the hypotheses formulate

Unit sales associated with the carry-over from the treatment of no price reduction with no point-of-sale promotion were greater than for any other treatment carry-over effect. The carry-over effect from the treatment of a price reduction with point-of-sale promotion is associated with the lowest unit sales. This is exactly the opposite from the direct treatment effect, where observations of unit sales increased as treatments progressed from T_1 to T_4.

This might indicate some stockpiling - i.e., the higher that sales are in one period, the lower they are in the following period in the same store because purchasers would buy more than their immediate needs.

An analysis of variance among all possible pairs of carry-over effects indicated, however, that no significant difference exists among pairs of carry-over effects on unit sales. For this analysis the carry-over effects were paired into the six different total possible combinations.

Four by Four Latin-Square Analysis : The advantage of the extra-period Latin-square is in studying carry-over effect. The disadvantage of the extra period design is that in each store one treatment appears twice. The extra period eliminates the advantage of equal store effect associated with all the treatment variables.

Since the treatment carry-over effects in this experiment can be assumed negligible, the analysis of the regular Latin-square design was performed.

Results of the analysis of variance of the four by four Latin-square experiment are shown in Table (iii)

Table (iii) : Analysis of Variance of Results of Latin-square Experiment.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio	Probability > F
Time Period	281.1875	3	93.7291	2.1393	.1965
Retail Store	283.1875	3	94.3958	2.1545	.1946
Direct Treatment	7641.6875	3	2547.2291	58.1393	.0001
Residual Error	262.8750	6			
Total	8468.9375	15			

The F-Ratio for the variation between periods' indicated that this variation could occur by chance alone about 20 percent of the time. In other words, little significant effect on unit sales of products could be attributed to the differences between time periods during which the experiment took place.

The F-Ratio for the variation between the stores could occur by chance alone about 20 per cent of the time. Again little significance could be attributed to this effect.

The third F-Ratio indicated the effect on unit sales of the treatments was significant ($p < .001$). The treatments made up of

the four possible combinations of price and point-of-sale promotion were significant on unit sales of the product during the course of the experiment.

Table (iv) shows the analysis of variance of the effects on unit sales of the three components of the direct treatment, price, advertising, and their interaction. All three components were found to be significant beyond the .01 level. Price alone significantly affected unit sales. Point-of-sale advertising alone was significant. The interaction of price and point-of-sale advertising was significant, in addition to the individual effects of price and point-of-sale advertising.

Table (iv) : Analysis of variance of components of Direct treatment effects

Source of variation	Sum of Squares	Degrees Freedom	Mean Square	F-Ratio	Probability F
Price	2376.562	1	2376.562	54.244	.0003
Advertising	4192.562	1	4192.562	95.694	.0001
Price and Advertising	1072.562	1	1072.562	24.481	.0026

The presence of this interaction suggests that when the price special and point-of-sale advertising were used jointly, the resulting unit sales were greater than could be accounted for by the single addition of the price and advertising effects (see Figure (i)).

FIGURE - 1 -

RELATIONSHIPS BETWEEN POINT-OF-SALE ADVERTISING, PRICE, AND UNIT SALES OF PRODUCT.

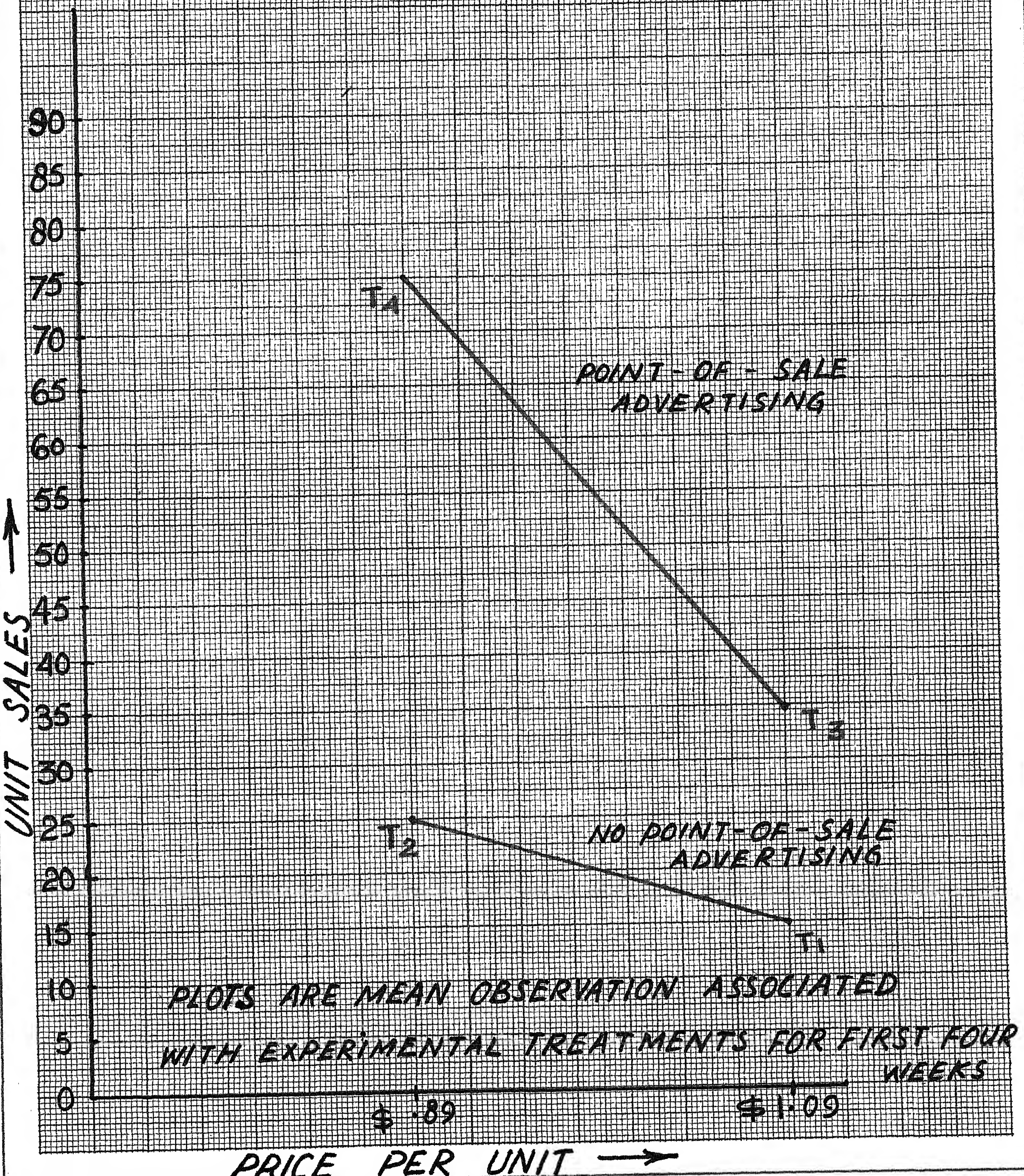


Figure (1) - Relationships between point-of-sale Advertising, Price, and Unit Sales of Product.

The tested hypotheses were based on the assumption that some effects on unit sales are associated with the experimental treatment combinations of price and point-of-sale advertising. The analysis of variance of the actual unit sales indicated that the experimental treatments were statistically significant.

A two-way analysis of variance was used in the study to investigate whether or not the treatments provided significantly different unit sales of the product.

The treatments were paired into the six different total possible combinations and the sales resulting in each of the four different stores.

The difference between each possible pair of treatments was analysed as to the unit sales of the product associated with each treatment in all of the retail stores used throughout the experiment. This was done since the hypotheses were formulated to compare the four treatments.

Differences among all possible pairs of treatment, as judged by associated unit sales of the product, were significantly. The two treatments of the pair $T_1 T_2$, neither of which utilise point-of-sale advertising, were significantly different at the .055 level.

All remaining treatment pairs were significantly different at the .02 level with the T_1T_4 pair and the T_2T_4 pair being significantly different at the .01 level.

Summary :- Consumers responded to point-of-sale promotion by purchasing more products than they did when a price reduction was utilized. Consumers did purchase more of the product when the price was reduced than when it was offered at the normal price. This was true when no point-of-sale or other promotion was utilized to call their attention to the product or its price.

When the consumer's attention was drawn to the product by means of point-of-sale promotion, there were more purchases than when no price reduction was used with the point-of-sale promotion. Consumers also purchased more of the product at a price reduction when point-of-sale promotion was used.

When pricing remained normal, consumers purchased more of the product when point-of-sale promotion was utilized. The combined effect of a price special and point-of-sale advertising resulted in more purchases than when neither point-of-sale advertising nor a price special was used.

When a price special was offered as incentive to consumer purchase behavior, sales increased. But consumers increased purchases more when offered the additional incentive of point-of-sale advertising than they did when the price special alone was offered as purchase inducement.

Carry-over did not have a significant effect on unit sales of the product in the experiment. The direct treatment variable was the only independent variable with a significant effect on unit sales of the product.

All three components of the direct treatment variable - price, advertising, and the interaction of the two - were found to be significant in effect.

When the effectiveness of a crude point-of-sale advertising device was compared to a price special, the results indicated the superiority of the point-of-sale advertising device in producing increased product sales.

A synergism is found to be present when point-of-sale advertising and price specials were combined. The independent increases in sales associated with the price special and point-of-sale advertising were 8 and 16 units, respectively. This indicates that if both were used together, the total effect would be the sum of the two independent effects, or 24 in this study. The increase found in this study when both point-of-sale advertising and a price special were used was 57 units.

Ambar G. Rao and Peter B. Miller[105] in their work on "Advertising/sales Response functions" studied bi-monthly sales of five level brands in 15 districts and related them to dollars spent on TV, print, price-off and trade promotions.

Allocating a national advertising budget among competing

geographical areas is a key decision that faces marketing management. Various rules of thumb are often used in marketing this decision, such as allocation proportional to sales or proportional to potential sales, or awareness or other indirect measures of sales potential. Since the real sales implications of such rules are poorly understood, they often result in allocations that are far from optimal from a sales or profit viewpoint.

A management science approach tothe allocation problem requires a more direct measure of the relationship between marketing expenditure (investment) and sales (profit). Rao's paper describes and illustrates a procedure for empirically estimating advertising/sales relationships from historical data. The procedure uses distributed lag models to relate market share to advertising and promotional expenditures on a market-by-market basis as a building block. The relationships are parametrised by geographical characteristics where necessary. The procedure has two steps :

1. The derivation of empirical advertising response coefficients forthe brand under study for each of a number of sales districts,

2. The synthesis of the empirical advertising response coefficients into a general model of sales response to advertising.

Development of time lag characteristics of advertising is an integral part of the method.

The procedure has been tested using data from a number of brands of consumer packaged goods. In every case the advertising/ sales relationship was successfully estimated.

After describing other approaches to the allocation problem, methodology, data, the results, and the application and extension of the findings are reviewed. In the subsequent discussion, absolute magnitudes have been scaled to protect the confidentiality of the data. However, the functional forms and scientific conclusions deduced are unaffected by this scale change.

Other approaches : Many different strategies have been employed in attempting to obtain a management science approach to the problem of allocating an advertising budget. They can be broadly classified into three groups : (i) controlled experimentation (ii) regression techniques, and (iii) decision models parametrized by subjective estimates.

An example of the first approach is the now-classic series of experiments conducted by Anheuser-Busch in the 1960's. The results obtained from such experimentation were, from a scientific viewpoint, the best basis for decision-making. In practice, however, proper experimentation required a high degree of management commitment over an extended period of time - something that is often hard to obtain, especially in multi-product companies where any one product and its advertising budget is not considered critical to the financial health of the company.

A second approach has been the use of historical sales or share data together with information on advertising expenditure levels in constructing regression models relating sales to advertising. The greatest success in this approach has been through the use of distributed lag models (Montgomery and Silk, 1970). Published examples have been restricted to long stretches of data (five years, for example) gathered on a national basis. Thus, the results cannot be used for geographical budget allocation.

Even when the distributed lag approach is used on a district-by-district basis, it is difficult to use the resulting models directly for an optimal allocation scheme. Each model is essentially a linear approximation of the sales response to advertising valid in a small range of expenditures similar to that used in model development. Thus, the effects of large changes in advertising expenditures cannot be correctly predicted by such models.

The importance of the non-linear effect of advertising on sales has led to the third broad type of models. This approach, exemplified by Little's ADBUDG (1970), directly develops decision models using subjective estimates of the advertising/sales relationship. Little proposes that the model be then adaptively adjusted to fit the real world. This approach has merit, particularly when no objective method of obtaining the advertising sales relationship and associated quantities is available. However, a model with a very large number of subjectively estimated parameters is prone to improve only slowly and after the fact.

Method: The method is described graphically in figure 1.

Figure 1 - An Advertising Response Function

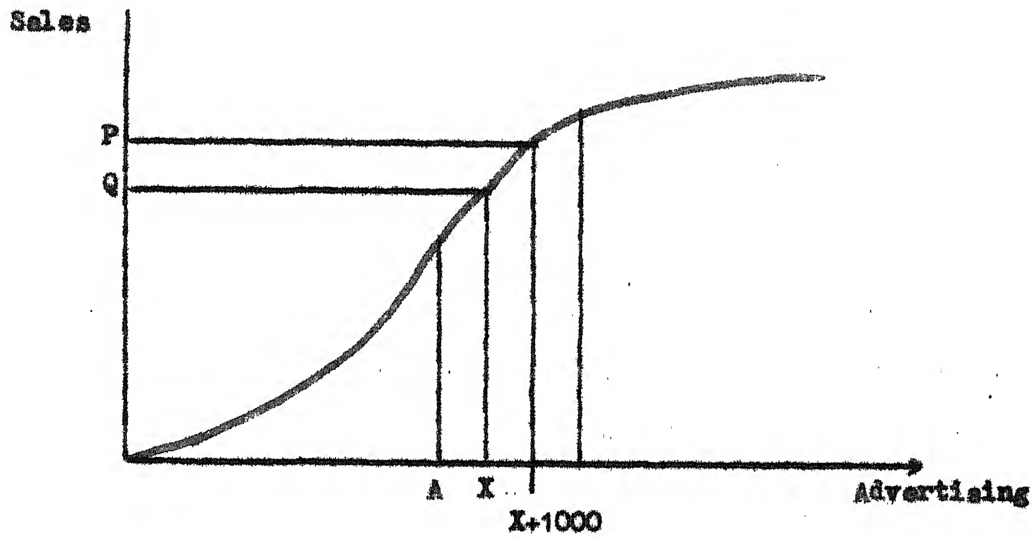


Figure 1 shows a hypothetical advertising/sales relationship. Consider a district where the average rate of spending per period has been X and where the expenditures themselves have been in the range (A,B). Given a rate of spending of X dollars, sales will be Q dollars. The incremental sales created by the expenditures of an additional thousand dollars is the quantity P - Q. If a distributed lag model is developed for this district, the quantity P - Q can be estimated. Note that the magnitude of P - Q depends on the location of the point X on the advertising axis. Thus PQ can be regarded as the first derivative of the advertising response function at X.

If models can be developed for a number of markets which

historically have had diverse levels of average advertising expenditure X, one can obtain estimates like P - Q at each of these levels of X. Thus, estimates of the slope of the advertising response function at various levels of X will be at hand. These are then synthesised to reconstruct figure 1. In practice, P-Q may depend not only on X, but also on market specific factors such as the brand's share of market.

One problem in statistical and econometric studies of advertising effectiveness is the separation of cause and effect, since companies normally allocate advertising funds as a fixed proportion of expected sales. Thus, a statistically significant relationship between advertising and sales might merely indicate that management was following its budgetary decision rule carefully. This is particularly likely when the time unit deoted for data gathering is about the same as (or an integral multiple of) that used by management for budgetary decision-making. For example, an excellent correlation is usually found between annual sales and annual advertising expenditure of a company - indicating only that the company has carefully followed its decision rules of budgeting advertising as a fixed proportion of sales.

The time unit problem can be avoided by selecting a sufficiently small time interval for data gathering. The smaller the interval, the less likely it is that management plans its allocation of advertising funds as a proportion of expected sales in that interval.

Counterbalancing this advantage is the increase in data collection costs as the time interval becomes smaller.

In this study, a reasonable compromise was to gather data on a bimonthly basis. This was consistent with the normal data gathering interval for the products studied and much smaller than the interval used for budget allocation purposes. There is no conceptual reason, however, why weekly or monthly data could not be used in studies of this kind.

The second decision concerns the time-span on which the models are based. In this case a two-year span was selected, again primarily because of data availability and cost problems. It should be noted that while intuitively a greater time span might seem desirable, it may complicate the model-building process rather than simplify it. Examples of difficulties encountered in longer time - spans are new products introductions, which alter shares in unpredictable ways, sales district redefinitions, and major changes in advertising themes by one or more of the brands.

For these reasons, all input data were bimonthly for 1971 and 1972. Fine brands were chosen for the analysis. Fifteen geographically distinct sales districts were selected for analysis of each brand.

Advertising expenditures were obtained for each district in terms of bimonthly dollar expenditures. They include network, spot, and print expenditures.

Promotion expenditures were obtained as follows :

Price-off Promotion: (deal pack) data was obtained from Nielson audit.

Other Promotions: referred to here as "trade" promotions were obtained from internal company data.

The actual procedure involved three steps :

Step 1. Model coefficients were estimated for each district relating market share to advertising, trade promotions, per cent of cases in deal, and per cent of competitive cases in deal. Since the chief goal was to obtain general conclusions about the effect of marketing variables on sales for a given brand, it was important to specify a common form of the model for each of the individual districts. Due to the limited amount of data available for estimation of model coefficients, several alternative forms of models could provide equivalent fits. Selection was guided by previous studies for frequently purchased, inexpensive products, and was consistent with studies published in the literature.

The basic assumption was that each marketing expenditure effect and also a lagged effect. This lagged effect was assumed to decay exponentially. Consider, for illustrative purposes, a district where advertising is the only marketing variables. Then the assumptions imply a model of the following form :

$$(1) \qquad S_t = C_o + C_1 M_1 + C_1 \lambda M_{t-1} + C_1 \lambda^2 M_{t-1} + \cdots\cdots + u_t$$

where,

S_t = share of market at time t

M_t = media expenditure at time t in thousands of dollars and C_o, C_1 and $\lambda < 1$ are constants

u_t = a random disturbance.

Equation (1) can be interpreted to mean that an incremental expenditure of one unit on advertising in a given period will yield C_1 share points in the same period, $C_1\lambda$ in the following period, $C_1\lambda^2$ inthe period after that, etc. The equation can be simplified by writing λ times the share of time t-1.

$$(2) \qquad \lambda S_{t-1} = \lambda C_o + \lambda C_1 M_1 + \lambda^2 C_1 M_{t-2} + \cdots\cdots + \lambda u_{t-1}$$

Subtracting (2) from (1) yields :

$$(3) \qquad S_t = C_o (1 - \lambda) + C_1 M_t + \lambda S_{t-1} + u_t - \lambda u_{t-1}$$

Rewriting (3), we have

$$(4) \qquad S_t = a_o + a_1 M_1 + \lambda S_{t-1} + w_t$$

A discussion of the statistical properties of the estimates under various assumptions about the w_t is found in Montgomery and Silk (1970). In this study, equation (4) is expanded to recognise three additional factors. The first is the effect of deals. In previous studies, immediate gains caused by deals were frequently found to be offset (partially) by losses in the next period. The second is the effect of trade promotions. If a trade promotion occurred in a given month, a value of 0.5 was associated with the Nielsen period of which the month was a part. In addition, it was

considered likely that in many instances, there might be losses in share in the period following a trade promotion. The third factor is the effect of competitive deals. Terms for the effect of competitive deals were included to account for an immediate and one period lagged effect. The full form of the model estimated for each district was :

$$(5)\quad S_t = a_0 + a_1 M_t + b_1 D_t - b_2 D_{t-1} + b_3 TP_{t-1} - b_4 TP_{t-1} - b_5 C_t + b_6 C_{t-1} + \lambda S_{t-1} + w_t .$$

C_1 = per cent in deal for competition at time t,

D_t = per cent in deal at time t,

TP_t = Trade promotion at time t.

As mentioned above, this model implies an exponentially declining effect of advertising. By recursively substituting S_{t-1}, S_{t-2} .... in equation (5), it can be shown that the exponential decay effect is being postulated for all types of marketing expenditure. The marginal short and long run effects of expenditure on advertising deals and trade promotion can be easily approximated from (5). The short term marginal effects are model coefficients a_1, b_1 and b_3 share points for unit increments in advertising, deals and trade promotion, respectively.

The long run marginal advertising effect is $\frac{a_1}{1-\lambda}$. The long run marginal deal effect is $(b_1 - b_2)/(1 - \lambda)$. The long term marginal effect of trade promotions is $(b_3 - b_4)/(1 - \lambda)$.

In the initial phase of model building, the model given by (5) was fitted to district data. Various ways of coding the trade promotions were considered, finally it was found that the best fits were obtained by considerations of three particular types of trade promotions and ignoring all others.

After the model had been fitted, the level of significance of each of the coefficients was tested. Terms with coefficients that were found not to be significant at the 90 / level were dropped (assumed equal to zero) and the model reestimated. When the coefficient of the S_{t-1} term was found to be non significant, a term in M_{t-1} was introduced in an effort to detect an alternate form of lagged effect of advertising.

Let I be the industry sales per year in the district, P the district population in thousands, and AV the average rate of advertising expenditure inthe time span used for model development.

In the discussion above, a $ 1,000 increase in advertising in a given Nielson period produces a cumulative incremental effect of $\frac{a_1}{(1-\lambda)}$ share points. Since industry sales per period is $\frac{1}{6}$, the total incremental sales generated per $ 1,000 additional expenditure on advertising per period is given by $a_1(1-\lambda) \times \frac{1}{6} \times \frac{1}{100} = S$

Let y = total incremental sales per thousand population for each additional $ 1 advertising per 1,000 population per period given average spending level $\frac{AV}{P}$ per 1,000 population per period

$$= \frac{\text{total incremental sales}/1000}{\text{additional \$ 1 advertising per period}}$$

$$= \frac{S}{1000}$$

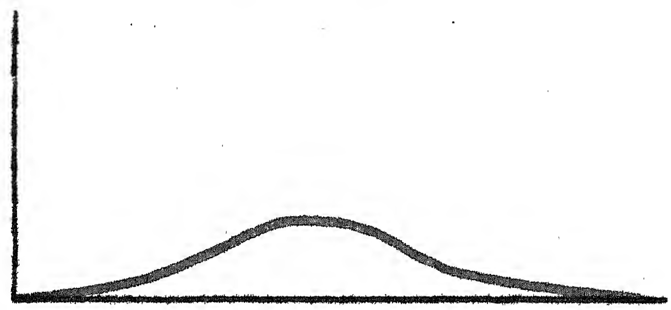
Thus for each district, y and the corresponding AV/P = x were computed.

Step 2 : Estimate the functional relationship between x and y. As discussed above, y is the derivative of the advertising response function when advertising expenditure takes the value x. In symbolic terms, if g(x) represents the advertising response function, then $y = \frac{dg(x)}{dx}$. Given that the advertising response function has an S shape, its first derivative should be approximately as shown in figure 2. The points where the curve becomes asymptotic to the x-axis correspond to the threshold value and the saturation value of advertising expenditure; its maximum corresponds to the point of inflexion of the S curve. Over most of its range, this curve can be fitted by a quadratic equation in x.

Although a substantial amount of the marginal response variation could in all cases be accounted for by the variation in average advertising expenditures from district to district, two other variables were investigated which a priori were likely to have explanatory power. The first was average share of district market. The second, which is highly correlated with share of market, was the share of premium-priced brands. The latter had the better explanatory power (in terms of improved fit) of the two. It entered

significantly into the models for several brands studied. In every case; including those where the relationship was not statistically significant, it effected the marginal response positively, that is, a given expenditure on advertising was more effective in a market with high share of premium-priced brands than one with a low share.

Figure - 2 - First derivative of the Advertising Response function.



It is probable that as share of market increases beyond the range encountered in this study, its effect on the marginal response to advertising will decline, and eventually become negative. That is, as brand share increases beyond some critical point, acquisition of further share becomes increasingly more costly. This hypothesis remains to be substantiated with data.

Functions of the form $Y = K_o + K_1X - K_2X^2 + K_3X$ were fitted to the marginal response coefficients, where :

X = Advertising $/1000 persons per Nielson period.

Y = change in sales $ /1000 persons per year for a $ 6
change in advertising $ /1000 persons per year

Z = per cent share of premium priced brands.

These functions approximate the form indicated in figure 2 over the part of their range which is important from a decision making viewpoint.

Step 3 : The functions estimated in Step 2 for advertising expenditure are integrated and the advertising / sales response function is obtained.

The procedure which has been discussed is slightly modified to obtain sales expenditure relationships for deals and trade promotions.

This procedure for estimating the advertising response function improves the models of the various districts in a very direct way. Once the various district models for a brand have been developed and their marginal response plotted, one or two markets usually seem to be "out of line". Since many alternative models of equivalent statistical validity can usually be fitted to a set of data, the existence of outliers signals the analyst that one of two things has happened :

1. an incorrect model has been specified and, therefore, an alternative model which provides an estimate of marginal response more consistent with the pattern formed by the other markets should be sought.

2. The inconsistency is due to some market-specific factor (unusually high market share, for instance).

Thus, all data are used simultaneously, albert in an informal way. Each district model is developed on the basis of data for that district alone, but is checked for reasonableness by comparison to all other district models for the brand.

Current research is being conducted by the author to formalise this simultaneous character of the estimation procedure. A promising approach seems to be the modification of the linear programming technique for estimating regression proposed by Wigner (1959).

From a decision-making viewpoint, however, the present approach is adequate, and it is not clear by any means that a simultaneous estimation procedure will improve accuracy in any significant fashion.

<u>Results</u> : Study results clearly indicate that the relationship of sales to advertising expenditure is approximated by a family of S-shaped curves. That is, there is a minimum level of advertising expenditure below which the marginal response to advertising is very small. For higher levels of expenditure, the marginal response is larger. As advertising expenditure continues to increase, the marginal response tends to become small again.

The marginal response to advertising ($ sales change/ $ advertising change) is exhibited in graphic form in figure 5 to

FIGURE - 3 - FIT OF THE MODEL FOR BRAND A IN DISTRICT-

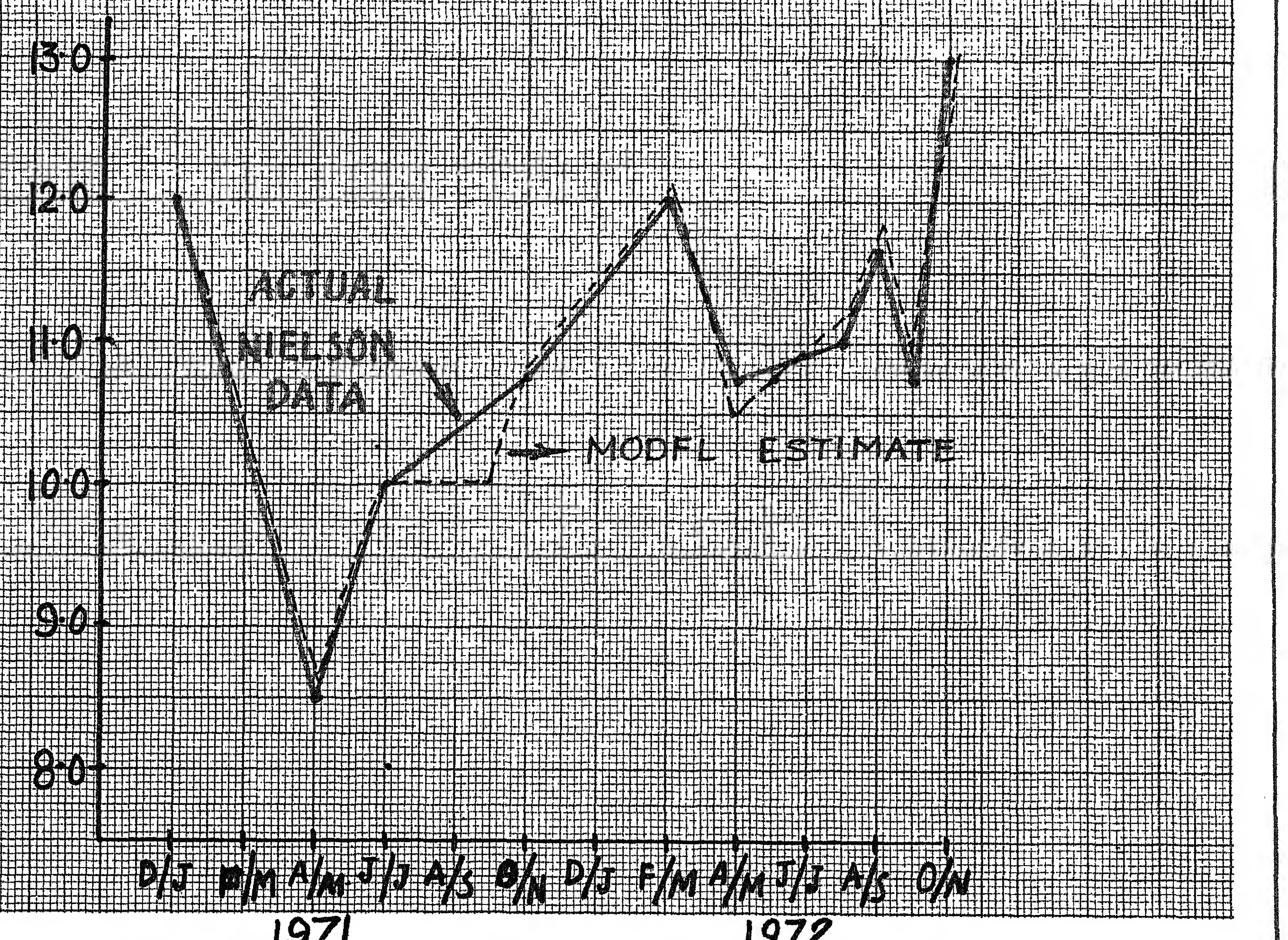


FIGURE-4-FIT OF THE MODEL FOR BRAND "A" IN DISTRICT-2

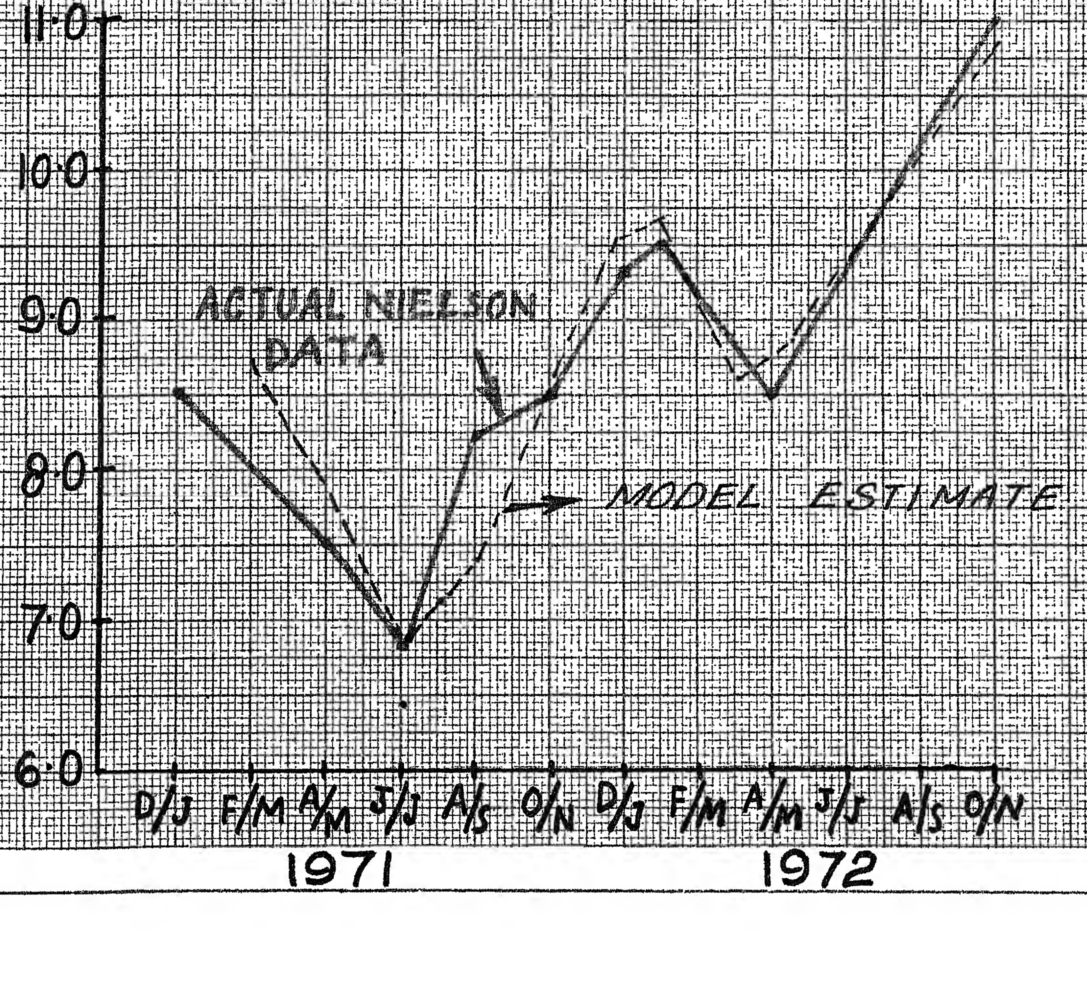
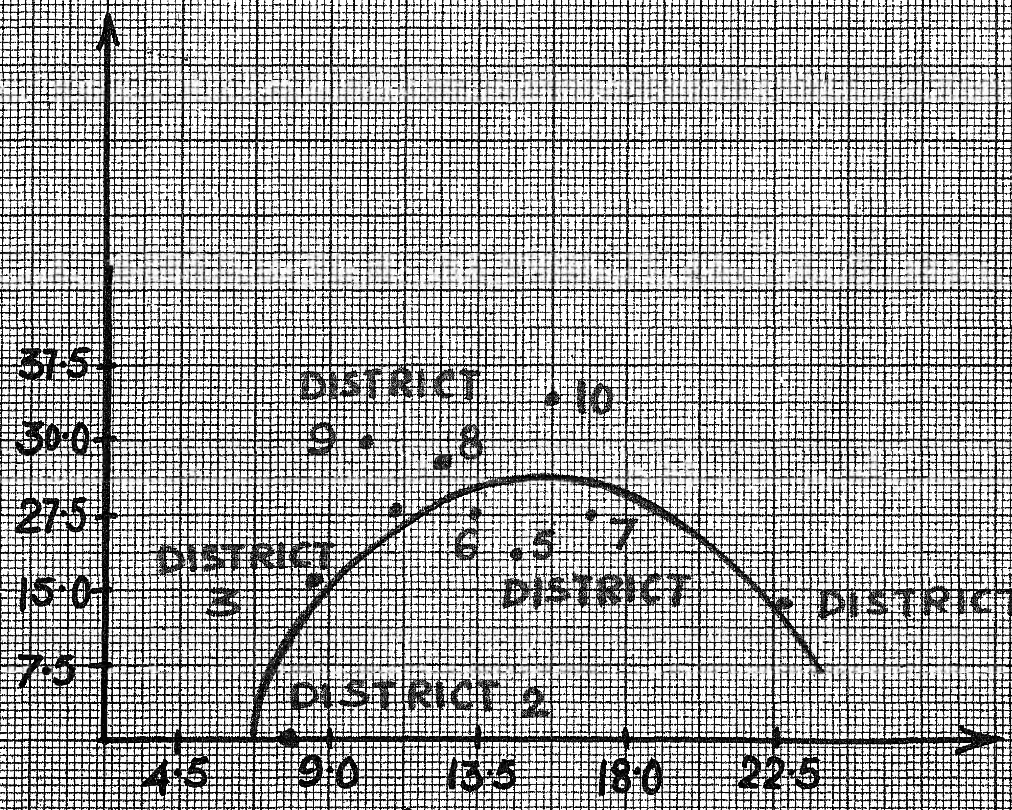


FIGURE - 5 -

RELATIONSHIP BETWEEN MARGINAL SALES CREATED BY ADVERTISING AND AVERAGE ADVERTISING EXPENDITURE LEVELS FOR BRAND B.



Y = CHANGE IN SALES $/1000 PERSONS PER YEAR FOR A $ 6 CHANGE IN ADVERTISING $/1000 PERSONS PER YEAR.

X = ADVERTISING $/1000 PERSON PER YEAR

illustrate results for brand B. This graph corresponds to figure 2, discussed in step 2 of the Method section. The dependence of marginal response on market share, discussed in the same section, is illustrated for brand E in figure 6.

Figure 3 - Fit of the model for brand A in district 1.

- - - - - - - - - - - - - -

Figure 4 - Fit of the model for brand A in district 2.

- - - - - - - - - - - - - -

Figure 5 - Relationship between marginal sales created by Advertising and Average Advertising expenditure levels for brand B.

The fit of the model estimates to the observed Nielson bimonthly market shares in each district are generally good, as shown by the average R^2 (Table 1). Some representative examples of the fit of the model estimates to observed market share are shown for brand A in districts 1 and 2 in figures 3 and 4.

The fits of the functions relating marginal response to average expenditure as described in equation (6) in step 2 were also adequate, the relevant R^2 are shown in Table 2.

Y = change in sales \$/1000 persons per year for a \$ 6 change in advertising \$/1000 persons per year

X = Advertising \$/1000 persons per year.

Figure 6 - Relationship between marginal sales created by Advertising and Average Advertising expenditure levels for brand E.

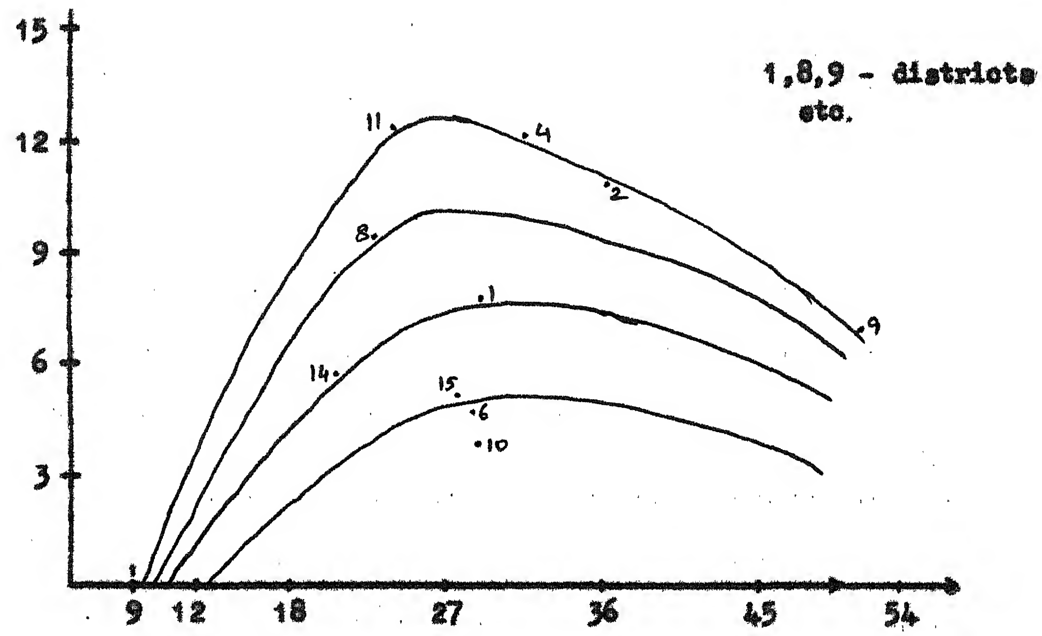


Table 1 - Average Values for R^2 for District models

Brand	Average R^2
A	.80
B	.74
C	.66
D	.67
E	.60

Table 2 - R^2 for estimated Relationship between marginal sales created by Advertising and Average Advertising expenditure levels

Brand	R^2
A	.62
B	.60
C	.63
D	.34
E	.80

The integrated form of the marginal response function shows the extra sales dollars generated for each corresponding level of advertising expenditures (see figures 7 and 8). The non-linear nature of the sales/advertising function is effectively demonstrated with both the threshold effect and eventual saturation clearly evident.

It should be noted that the vertical axes in these graphs represent extra sales dollars achieved in a district by advertising above a large base sales level. The base sales level may be thought of as the sales level which is present in a district when no advertising expenditures are made. Such a base sales level is, of course, dependent on other brand and demographic characteristics in the district.

Figure 7 - Advertising Response function for brand B.

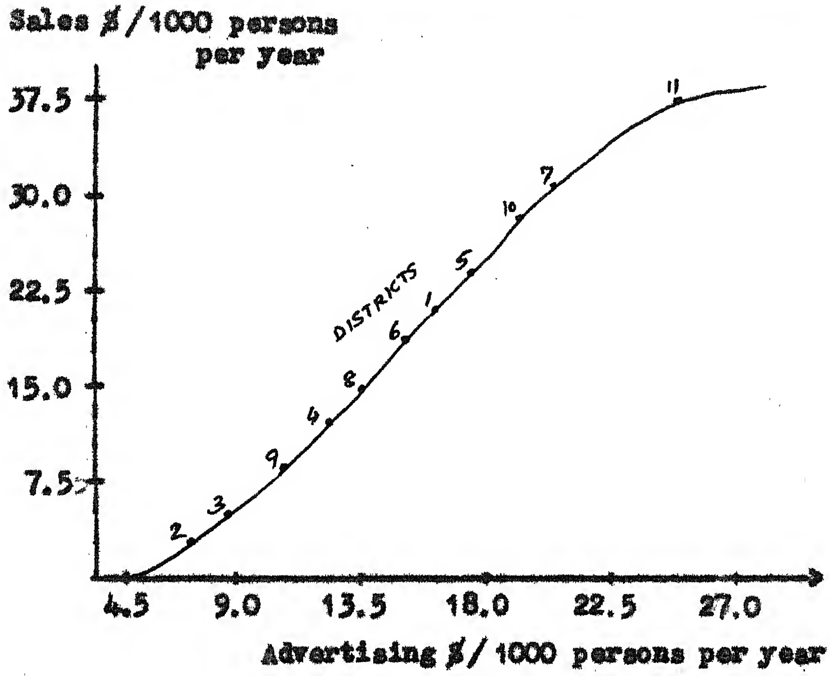
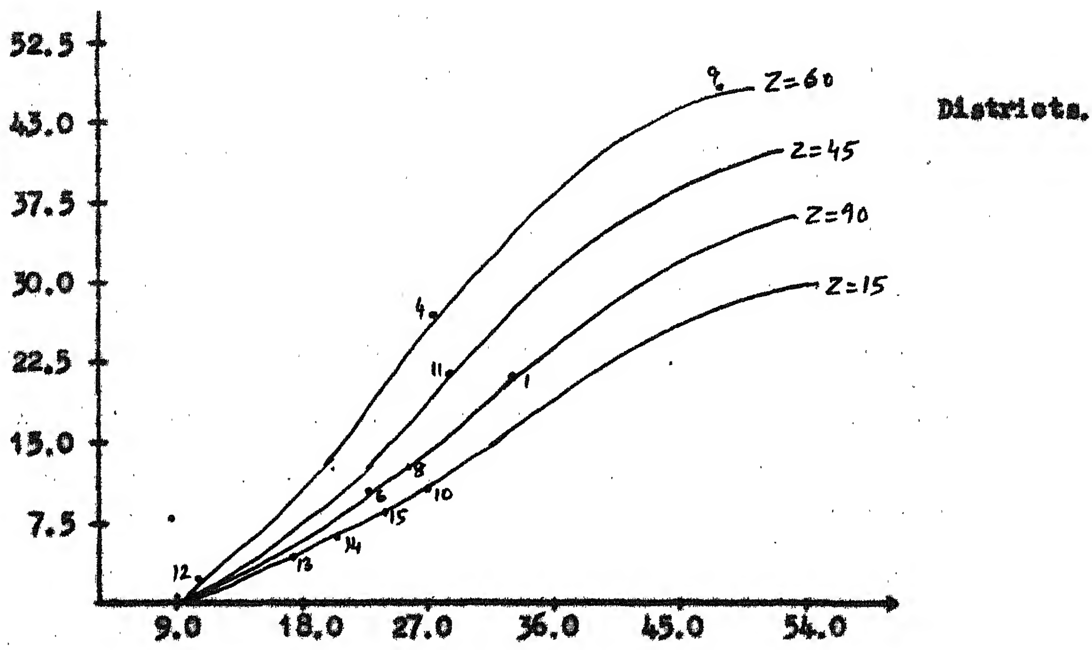


Figure 8 - Advertising Response functions for brand E.



Applications of Advertising/sales Response functions :

The advertising/sales Response functions developed in this study can be used for :

1. District advertising Leavy-up tests and national projection.
2. Evaluation of sales/profit consequences of alternative advertising budget plans.
3. Allocation of given national budgets across districts in an optimal manner.

District Advertising tests and National Projection: Brands frequently conduct advertising heavy-up tests in which ad expenditures are increased in a selected district in an effort to determine the potential of a national increase in advertising weight. The advertising/sales response functions can supply this information so long as the marketing environment remains stationary.

In case it is felt that conditions have changed since the periods for which the response function was estimated, it can aid in planning better heavy-up tests and in turn can be updated by the results. Prior to a test, the response function permits : 1. Identification of districts which have the greatest potential (in terms of sales or share) response to the planned increase in advertising expenditure. 2. Prediction of these responses.

Market share and sales measures usually employed by brands are based on audits of samples of stores. Thus, they are subject to

sampling variation. Selection of a market which is likely to have a substantial sales response increases the possibility that the effect actually observed will exceed sampling error significantly. It also reduces the cost associated withthe test.

Once the heavy-up has been completed, the actual result can be compared with the prediction and the response function adjusted if necessary. This function can then be used to predict the consequences of national increases in advertising expenditure.

Evaluation of Sales/Profit consequences of Alternative National Advertising budget Plans : Evaluation of sales and profit consequences assuming a constant bimonthly rate of spending of different national advertising budget plans for the five brands can easily be made using the advertising response function obtained for the brand.

Allocation of Given National budget across districts in an optimal manner : Allocation here means both in terms of dollar advertising expenditures for district and timing of advertising on a bimonthly basis (e.g., pulsing). The individual district models provide information about the lagged effect of advertising expenditure and the duration of these lags.

Table 3 illustrates the application of the model to a particular heavy-up set.

Table 3 - Advertising Heavy-up tests : Brand C - Major Market X

Base period : Feb./March 1971 - Aug./Sept. 1971 Test period Oct./Nov. 1971 - April/May 1972

Actual major market Results :

Base period Advertising $ Expenditure (Annual basis)	Test period Advertising $ Expenditure (Annual basis)	/ Advertising Increase	Increase in market share due to heavy-up
$ 129.75	$ 264.15	1.03	0.8 /

Results predicted by Advertising Model

$/1000 persons per year	$/1000 persons per year		
$ 17.85	$ 36.15	1.03	0.6 /

The Actual and predicted results have been scaled by the same factor for confidentiality, but ratios are not affected.

In Table 4, two alternative policies, one constant spending rate per bimonthly period and one of pulsed expenditure, are compared to illustrate the importance of considering not only allocation of among districts but allocation over time. It is clear that a computerised system for planning advertising expenditures may be developed, applicable to all brands for which results similar to those obtained for brands A, B and C are available.

Table 4 - Illustration of Pulsing for brand A

	Market	Policy 1	Policy 2
Incremental sales/1000 persons	1	1	9
	2	6	12
	3	4	11.5
Total Incremental sales	1	5,800	52,200
	2	57,600	115,200
	3	58,000	166,750
Sales given by advertising / cost of advertising	1	.08	.73
	2	.35	.71
	3	.27	.77

Policy 1 = Constant spending rate

Policy 2 = Pulsing every alternate period.

(Table 4 shows the improvement in advertising effectiveness produced by varying the rate of expenditure. In all three markets existing budget allocations are close to the threshold levels; these allocations are being spent at an approximately constant rate per time (Policy 1). An alternative policy (although not necessarily an optimal one) is to maintain the same annual budget in each market as for Policy 1, but to advertise only in alternate Nielson periods (Policy 2). This policy more than doubles the sales/advertising ratio, although, strictly speaking, it is still improfitable to advertise at these annual rates).

This system can be designed to have the following capabilities :

1. The system will be able to optimise the allocation of advertising budgets across districts and over time.

2. It will be able to compare alternative proposed plans and evaluate them ona sale/profit criterion.

3. It will be able to allocate a given division bueget across brands in an optimal manner (e.g., brands A, B and C).

4. Various types of summary reports will be obtainable from the system.

Thus, the method of synthesising the results from separate distributed lag models for each of many districts has enabled the construction of empirical non-linear functions of the sales/advertising response function for a particular brand and makes possible a wide variety of practical applications.

Curhan[106] conducted research on the effects of merchandising and temporary promotional activities on the sales of fresh fruits and vegetables in supermarkets. This research was supported by funds from the COSMOS project managed by case and company for the National Association of Food Chains and the Marketing Science Institute. Retail price, newspaper advertising, display space, and display location quality were tested at two levels for selected fruits and vegetables according to a fractional fractional research design in four large supermarkets. The resulting impact of sales rate was analysed for four classes of items; hard fruit, cooking vegetables, salad vegetables, and soft fruit. The effect of advertising was found to be significant for hard fruit and cooking

vegetables. Advertising has a negligible effect on sales of salad vegetables and soft fruits. This suggests that while the purchase decision in the former two categories is influenced by advertising, the purchase decision for salad vegetables and soft fruit essentially is an in-store decision based on the appearance, quality, and value of the product. This is not to say that the cumulative effect of advertising soft-fruit and salad vegetables does not make a valuable contribution to store image. This experiment does suggest, however, that sales of produce staples respond more directly to advertising stimuli than do discretionary purchase items.

Table 1 - Percent increase in unit sales for "high" levels of Advertising

Product characteristics	Hard fruit	Cooking Vegetables	Salad Vegetables	Soft fruit
Average (main effect)	33[a]	89[a]	5	3
High volume	28[a]	71[a]	18	12
Low volume	38[a]	107[a]	-8	-6
High price	50[a,b]	116[a,b]	-8	-10
Low price	16[a,b]	62[a,b]	18	16
Seasonal	39[a]	128[a,b]	-10[b]	2
Non seasonal	27[a]	50[a,b]	20[b]	4

[a]Main effect significant at the .25 level or better.

[b]First order interaction significant at the .25 level or better.

The effect of advertising is positive and significant both for high- and low-volume hard fruit and, especially, for cooking vegetables, although the impact is greater for slow-selling products in these categories. Displays of low-volume products ordinarily are dwarfed by larger displays of fast-selling products within these categories. Advertising probably serves to call attention to these otherwise "invisible" products.

Results by price also are mixed. The reported effect of advertising was greater for high-priced products within the hard fruit and cooking vegetable categories, but still positive for low priced products. The reverse tends to be true for soft fruit and salad vegetables, although the effects are not significant.

The effect of advertising was very large for seasonal products in the cooking vegetables category, although significant and positive for nonseasonal cooking vegetables and for seasonal and nonseasonal hard fruit as well. Cooking vegetables are relatively "invisible products" for which advertising probably serves to attract consumer attention.

Care must be exercised in drawing conclusions about the main effect of advertising from the results of salad vegetables, since this is another instance where, although the estimate of the interaction is significant, the main effect is not. The finding that advertising is not significant either for seasonal or non-seasional soft fruit again supports the theory that customers "shop" this department, rather than depend on advertising as a major

source for purchase information.

Certain findings of Curhan's study may have immediately applicable operational importance to supermarket managers. For example, perhaps advertising effort should be concentrated on hard fruits and cooking vegetables and withheld from salad vegetables and soft fruits.

Krugman[107] is of the view that through thorough analysis of audience psychology, the advertiser is better equipped to avoid too much or too little exposure.

Any discussion of the effects of advertising must begin with the environment - a controlling factor that both defines and limits the possible effects. The environment of advertising consists of an intricate web of social, economic, and technological circumstances that direct an ad toward a particular audience through a particular medium obviously, the advertiser's primary concern in this advertising environment is the consumer himself. How much attention does he pay to the advertising that surrounds him? An answer to this question began in 1922 when a pioneer psychologist, Daniel Starch, devised a method for measuring reader's recognition of print advertising. In 1932 he established a readership research program that, through more than 240,000 interviews, annually surveys the readers of more than 1,000 issues of consumer and farm magazines, business publications, and newspapers.

To provide a reasonably precise answer to the question of how many people notice and read an advertisement, Krugman chose the most

common type of print advertising - the one-page, four-color advertisement - and totaled the starch results for all such advertisements (20,347) in all issues of 47 major magazines in 1970. Krugman found that 44 / of readers claim to have noticed a particular advertisement and 35 / read enough to identify the brand, but only 9 / say they read most of a particular advertisement. In other words, almost half of all advertisements are noticed - a third to the point of brand identification - but less than a tenth are of enough interest to be read. Naturally, the responses vary depending on ad size, content, and position, on the receptivity of the reader (sex, age and income), and on whether the reader is in the market for the product advertised. At any rate, only a small portion of advertising is fully perceived at any time.

The situation in television is similar. In the 1960's, George Gallup, another psychologist who pioneered in media research as well as in public opinion research, instituted a survey in which, for example, a cross section of Philadelphia was telephoned the day after an evening of television and asked to recall the commercials on the prime-time shows. On the average, only 12 / of those who had seen a particular program could recall its commercials.

It has been demonstrated in many ways that people filter out much of the huge quantity of advertising to which they are exposed. For example, in a study conducted in 1968 by the American Association of Advertising Agencies and Harvard University, 1,536 participants,

representing a national sample, were equipped with counters and asked to register every advertisement they saw. Each person enlisted for half a day. The number of exposures per person fell more often into the 11-20 category than into any other, which indicates that the respondents perceptual screens were very effective indeed[108].

The readers of advertisements are shopping for information, and they are aware of much product publicity that they hear and see. But they fully absorb and perceive only those portions which interest them. These research findings on attention help define the nature of advertising's very special and restricted ability to persuade.

In view of this limitation, it may be useful to discuss what can be expected of advertising - that is, what processes produce what effects. It may also help to spell out the differences in impact among one, two, and multiple exposures to an ad. Finally, it may be helpful to show how to use industry figures to avoid use of too little or too much advertising.

There are three ingredients of successful advertising : information, rational stimulus, and emphasis. Few advertisements boast all three, but a good advertisement possesses at least one skillfully treated characteristic.

Information is the simplest ingredient. If your message contains news the advertisements skills brought to bear on it are

not critical to the advertisement's success. The agency that first reported the American DentalA Association endorsement of Crest toothpaste had a rather easy job in producing an effective ad because the nature of the information almost guaranteed success. This is not a common phenomenon, but manufacturers of new products hope that their brain children will be accorded similar receptions.

Rational stimulus is the ingredient that provokes the consumer to evaluate, judge, and reach a decision. This response obviously is most common when the consumer happens to need or want the product or service in question. Involvement with the message is a prelude to, a substitute for, or a supplement to contacts with salesmen and stores. This rational process usually characterises the reaction to ads for more expensive products and services; in such cases, an unwise purchase will haunt the shopper longer than if he had made a mistake buying a can of peas.

With the third ingredient, emphasis, the matter becomes more complex. Emphasis is particularly important in connection with less expensive products, products of relatively little importance to the consumer, and products with few differences among the brands represented. Consequently, the consumer is less interested in the advertising and is more likely to screen it out. The advertiser in such cases is more likely to emphasise a single theme or one aspect of the product. Also, he repeats frequently to gain attention and make his message familiar to the public. This, the most difficult type of advertising, seems to be the most appropriate for television

and its so-called captive audience. In addition it contributes the most irritating and the most entertaining commercials of all. It also, however, receives the most criticism from citizens concerned with taste. The amount of advertising and media effort in comparison withthe seemingly modest increments in brand preference seems disproportionate to many observers, who charge that the effort is wasteful. Or, at the other extreme, they claim that the endless repetition of simple themes produces in consumers a witless compulsion to go out and snatch the product - whether they need it or not.

The fact, is, however, that inexpensive and, what seem to some, trivial products also have to be purchased. For many people, such products do not merit enough concern to require involved comparison shopping; rather, /consumers put them with little reflection into their shopping carts. The advertiser, who cannot hold a long discourse with the shopper when his advertisement is seen or heard, can only hope that a residual effect will still be operating at the time of purchase. In short, advertising by emphasis aims for small, delayed effects, points that stick inthe mind long enough to tip the scales in favour of brand A over brand B.

The exaggerated power that some critics attribute to repetitive advertising is based on an assumption that the consumer is being manipulated against his will by the message and "programmed" to buy the product immediately. It is not generally recognised that the advertising is designed primarily to produce an effect that

persists after perceptual screening and forgetfulness have taken their tool. Most advertising has the modest goal of capturing attentionand maintaining awareness; outright persuasion is a secondary consideration.

Krugman next tackles the question of how much advertising is enough. A lot of money is spent on repetitive advertising. Some experts explain this by noting that recall of an advertisement drops without constant reinforcement. Others note that members of the audience are not always in the market for the advertised product, but when they are, the advertising must be there. There is no choice but to advertise frequently.

Krugman argues against single-exposure potency and also against much-repeated exposures. The differences between the first, second and third exposures. The differences between the first, second and third exposures have to be understood. Krugman is of the view that, psychologically, there is no such thing as a fourth exposure; rather fours, fives and so on are repeats of the third exposure effect.

The importance of just two or three exposures, compared to a much larger number, is attested to by a variety of research methods yielding similar research findings[109].

The first exposure is by definition unique. A 'what is it?' type of cognitive response dominates the reaction - that is, the audience tries to understand the nature of the stimulus. Anything

novel, however unattractive it may be on second exposure, elicits some response the first time, even if it is only the mental classification required to discard the object as of no further interest.

The second exposure to a stimulus, if it is not blocked out, produces several effects. One may be the cognitive reaction that characterised the first exposure, if the audience missed much of the message the first time around. This is most likely if the medium is Radio or television, where the tape or film cannot be rewound or reversed. More often, an evaluative 'what of it?' response replaces the 'what is it?' response, appreciating the nature of the new information, the consumer wonders whether it has relevance to him. If he absorbs the advertisement during the first exposure and finds it interesting, some of the 'what of it?' response may take place then.

Another element of the second exposure, and unique to it, is the recognition response "Aha, I've seen this before!". Such recognition permits the viewer or listener to pick up where he left off, without the necessity of repeating the cognitive step "what is it?" So the second exposure prompts an evaluation and, consequently, the "sale" occurs. This 'what of it?' response completes the basic reaction to an advertisement or commercial.

By this time the consumer is familiar with the message and the third exposure constitutes a reminder, if a decision to buy based on the evaluations has not been acted on. The third exposure is

also the begining of disengagement and withdrawal of attention from a completed eposode.

Krugman suggests that this pattern holds true for any multiple exposure. Most people filter or screen out TV commercials, for example, by stopping at the 'what is it?' response. But these same people, suddenly in the market place months later for the product in question, may see and experience the twenty-third exposure to the commercial as if it were the second- that is, the twenty-third exposure will be only the second time it really commands their attention.

Then the viewer is ready to absorb the message and evaluate it in terms of his or her needs. The twenty-fifth exposure, if not the twenty-fourth, will finish the sequence, subsequent exposures will arouse no further reaction. The viewer may still react to the commercial as entertainment or as an irritant, and while such a reaction may affect attitudes toward the advertiser, the industry, and the medium, these attitudes are probably irrelevant insofar as the response to the commercial message is concerned.

Krugman concludes that the effects of advertising on an individual are modest, but they are powerful in the mass and over time. Good advertising gets attention, and its message lasts long enough through a few exposures to make one or two points. Optimally, these exposures should reach the target audience with an effective balance of a few exposures to most persons in the audience, rather than one exposure to many and many exposures to a few. Spillover

exposures to nontarget audiences should be minimal. Ideally, the target audience should be exposed when it is in the market or during known shopping periods. Advertising management needs precise information about audience attention and about patterns of reading, viewing, and shopping. But it is not enough to have great ads and fine media vehicles in which to place them, a good plan based on research is essential.

Zufryden[110] has done important work on the dual optimization of Media Reach and frequency. An integral part of advertising decision making is media selection. An optimization model is developed to study the impact of the dual objectives of reach and frequency maximization in the problem of media selection. The approach used is based on a nonlinear integer mathematical programming model. This model maximizes reach subject to a parametric frequency constraint. Model solution is then accomplished through a dynamic programming formulation. Other useful applications of the model are also described. More specifically, it is shown how this approach can be applied to media selection under (1) maximization of either reach or frequency, or (2) minimization of total costs subject to stated reach and frequency goals. Moreover, the model allows for the consideration of qualitative media factors.

Numerous media models have been described for the purpose of selecting optimal media mixes. These models have displayed

several alternative types of objective functions. The early models have concentrated on objective criteria directly related to either reach, frequency, or total exposure. Gensch[111] has reviewed media selection literature. Some of the more recent analytical models such as those proposed by Little and Lodish[112] and Zufryden[113] have used higher-level objectives, such as sales-maximisation.

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PART III

MATERIAL AND METHODS

MATERIAL AND METHODS

Basically, there are three approaches to sales evaluation of specific advertisements and of entire advertising campaigns.

The first method is a direct questioning approach, in which customers are asked to report the influence of advertisements on their purchases. A second approach attempts to relate product purchases or ownership to advertising exposure or expenditure, and this includes the application of mathematical analysis and the technique of correlation. A third method is the application of controlled experiments to limited markets or areas as a means of estimating later sales impact on the total market, and sales area tests are discussed in the latter group.

THE FIRST METHOD - Interviewing Customers :

There are many reasons why it is difficult to get comprehensive information from customers which will reveal the influence of specific advertisements on their purchases.

One obstacle which we immediately encounter is that most people believe they generate their own decisions and will actually deny the influence of advertisements. Social barriers to the

admission of certain influences have arisen, and it has become popular to disavow the acceptance of advertising. Since the accumulated advertising contribution to any single purchase may go back over a period of years, it is understandable why direct probes of the consumer's motivation have tended to be so fruitless.

Much advertising expenditure is assumed to help hold loyal customers, but the effects are more dynamic when people are induced to try new products or brands. Consumer-purchase panels, wherein families report a continuous record of purchases, provide information on changes of brands. In an investigator is quick to follow up brand changes with an interview, there is increased likelihood of learning the underlying causes of change. Even under these favourable conditions, direct questions about the influence of advertising on the purchase of small items have revealed little to justify advertising costs.

George Gallup, whose contributions to recognition and recall research are recounted in chapters three and four, has reported success with a different approach for obtaining testimony from new buyers of brands and products[114]. Women in a probability sample of households were asked in detail about circumstances surrounding recent product brands purchased for the first time. They were asked to try to recall where they got the idea leading to the new purchase. Questioning goes from the purchase to the advertising, instead of the more traditional method which tries to go from the advertising to the purchase. Advertising is treated as a cause for

action, and hence the name Activation Research, as used by George Gallup.

Much of the Gallup and Robinson activation interview is devoted to documentation of the initial testimony as to advertising effect, Respondents are required to recall what the advertising looked like and said and where it appeared. If television was the medium, people are probed for program name, date, content, and other means of verification. Physical proof of the possession of products is also requested. If a product is missing from the shelf because it has already been consumed, the housewife is asked to give a detailed account to support that fact. Elaborate inventories, especially of pantry and bathroom articles, are sought in order to issue maximum coverage of possible purchases of all related products.

Activation surveys lead to the development of an index number for each reported product brand; and the relationship between index numbers and sales is studied. There has been notable success in tracing brand switches and new product introductions, especially where heavy television advertising has been used. Results for magazines and other national media are also reported.

According to George Gallup, more recent activation experiments have aimed at measurement of advertising influence on three major categories of customers. Not only is the new brand buyer studied but also the repeat buyers and those who fluctuate in their choices of brands. Mathematical analysis is applied to the survey data in

order to establish the weights of these three categories in evaluating the total advertising effort.

While Gallup and Robinson have reported a considerably higher level of advertising effect by these methods than have other investigators, people in the advertising industry have been somewhat cautions in their acceptance of the method and the findings. It is difficult to assess the promise and the value of consumer testimony on the sales influence of advertising. One basic objection is that the method seems to measure change in buying, but if a woman has been buying Kellogg's Corn Flakes (for example) for her family for a good many years, she is not nearly so likely to answer that she was influenced by the advertisement to buy this product as would another woman who is buying the product for the first time.

Most research investigators in this area of buying behaviour have given up before finding sufficient evidence to justify more than a small part of advertising costs. Others have obtained such a disproportionate advantage of broadcasting over other media as to suspect their own findings.

Certainly television is a dramatic medium, and when a commercial message is closely associated with a program, it is relatively easy to recall and identify it in an interview. High television scores may reflect poor interviewing technique, but may also simply reflect the strength of the broadcasting medium.

Paul F. Lararsfeld of Columbia University has expressed a strong belief that interviews made with customers can identify those who bought a product brand as a result of advertising or of advertising in a specific medium[115]. It would first be necessary through a process of detailed questioning to bring out references to advertising influence and then to trace and confirm each influence. This means that the respondent would have to be capable of recalling advertising impressions. Lararsfeld has recommended a procedure for actual gathering of sales data or experimenting. His plans call for comparison between people exposed and not exposed to advertising, for application through comparative evidence obtained by treating major sales territories differently, and for mathematical analysis of the data. All these factors will be brought into the discussion inthe next section.

THE SECOND METHOD - Correlation of Advertising and Sales :

While most advertisers find it extremely difficult to measure the sales impact of specific advertisements, it is comparatively easy to obtain evidence of the exposure or penetration of their advertising messages. It is also possible to find out what brands of products people have in their homes. These types of evidence and the manufacturer's own knowledge of advertising expenditures and product sales may be studied to see whether there are logical relationships between advertising efforts and sales responses.

National advertising typically operates over extended periods and in combination with many types of promotional activity. The simple relationships between advertising cost or opportunity and accomplished sales can seldom be taken as proof that specific advertising caused the observed sales. Nevertheless, the correlation between advertising exposure and product possession is a useful type of evidence, when interpreted with caution. In this context, correlation refers to relationships varying from those which can be directly observed in the data to those requiring complex mathematical analyses of many variables. Precautions need to be stressed as much as procedures.

Since media are responsible for providing advertising with opportunities for exposure of ideas for prospective buyers, media have a considerable incentive to provide evidence of sales effects One of the earliest correlation studies was made - although not published - by a major magazine. Readers and non-readers were compared on possession of products advertised in the publication. More importantly, people claiming to have seen from one to five advertisements in a campaign series were compared with issues readers who claimed to have seen none of the advertisements. In almost every instance, those reading the issues and those seeing one, two or more of the advertisements showed progressively more product possession than non-readers of the issues. The gains were extremely favourable in relation to advertising costs.

When it comes to assessing increases in product ownership as direct effects of seeing or reading advertisements, one basic question

has to be settled. As compared with the extent that people buy products because they have read advertisements, to what extent do they read advertisements because they have already purchased the product? It is a common experience, especially after having made an important brand choice, to feel a heightened interest in advertisements of that brand.

Actually, a high correlation between product purchase and advertising readership may result from either of two kinds of cause-effect relationships - readership may help to lead to purchase, or purchase may help to give rise to readership.

High correlation coefficients may also result from a third kind of causation - probably the most important of the three possibilities. If a person is interested in and "sold on" a given product (or service) for whatever reason or reasons, he is inclined both to look for the product (or service) and for advertisements about it simply because he is interested in the possibility of purchasing it. Thus, there would be a high correlation between the eventual purchase of a product and readership of the advertising, since both kinds of behavior are concomitants of being interested in and sold on the product. In other words; both purchase and readership may be the results - without either being the cause - of some of these high correlations.

There is far less likelihood that new purchasers of particular brands will give attention to product-sponsored broadcast programs

than they will read printed advertisements because of recent product purchases. This may explain the long procession of correlation studies in the field of radio and then television, with their implied measurement of advertising effects. Broadcasting media have sometimes claimed direct sales evaluation and have on other occasions, made more modest claims of attitudes built up through communication of advertised ideas. Two decades of such correlation surveys have met with skepticism and often rejection on the part of technically trained research people.

An example of a simple division of viewers and nonviewers of sponsored broadcasts, where product possession is also known, is presented in Table 1.

<u>Table 1</u>. Percent of Viewers of Sponsor's Program Possessing brand

Brand and product	Frequent Viewers	Occasional viewers	Nonviewers
R brand of razor blades	11	8	7
M brand of mouth wash	13	11	8
T brand of Toilet soap	27	22	19
C brand of cake mix	21	18	16
B brand of breakfast cereal	9	7	4

These typical margins of difference in product ownership at first appear to be exclusive outcomes of advertising on particular

programs, but internal analysis of the survey data may reveal that brand ownership is only one of the differences between viewers and non-viewers of a sponsored program. The nonviewers may have less money or less need and may be further from store outlets. Many other factors may contribute to the situation.

Anyone who believes in the power of advertising usually assumes that some sales advantages reported in correlation studies are genuine advertising effects. The great problem is that of controlling the influence of variables, many of which are probably not even known. Investigators have tried to match samples on many characteristics, including exposure to all major media..The advent of television, with stations opening up one market at a time, permitted additional controls based on observations before and after stations were built. This provided more of an experimental design, disturbed chiefly by the fact that television stations came first to the most promising markets.

Following are a few of the recognized factors which should be considered when interpreting correlations of television advertising penetration or product possession among exposed and unexposed population groups.

The broadcast signal may be better for homes in areas where the product is also most accessible.

Viewing a program usually involves staying at home, and this same tendency may favour product use.

Size of family is one factor related to both program viewing and usage of many products.

Viewers, as individuals, may normally be faster or slower consumers of the advertised product.

Advertisers usually try to advertise their products by using media which are most likely to reach people who are most likely to buy the products. A correlation of advertising exposure and product use may simply reflect a degree of success in selecting media on this basis.

Nonviewers include many types of nonprospects, such as those too poor for the sponsored products.

The early owners of television sets or color sets are venturesome or prosperous people who may be more responsive to advertising of all kinds.

The ownership of a product brand should make it easier to remember both the brand and the program which it sponsors.

People who are 'inclined to make excessive claims of product possession may also tend to overclaim familiarity with programs and with advertising messages.

Despite the competition of various media for audience time, the viewers of a particular program may be "consumers" of more other media than are non-viewers.

When manufacturers increase their media expenditures to buy

television in selected markets, they may stop up the whole program of sales promotion at the same time.

When comparisons are made before and after television stations or programs enter a new market, there are opportunities for independent changes in the responsiveness of the market during the measuring interval.

While many of the above variables can be controlled by matching survey samples, this process often destroys the projectability of the results to a whole population.

The problem of matching samples and establishing a defensible basis of comparisons between exposed and unexposed populations has been dramatically attacked by Irwin M. Towers, Leo A. Goodman and HansZeisel in a proposal to measure the effects of nonexposure[116]. They would simply draw two parallel random samples of the population. Each sample would include the normal percentage of viewers of a particular program. One sample would continue without change, while the second sample would be denied exposure to the program and its commercials. Just how to control this non-exposure is, of course, a problem; but the two samples would then be comparable except that one would no longer be exposed to the particular advertising messages. The influence of non-exposure could thus be measured under scientifically comparable conditions. No other published proposal has heretofore provided for truely comparable test-groups, but it should also be said that the proposed study is

theoretical at this time and has not been carried out.

Media have not been alone in the attempt to establish meaningful correlations between specific advertising efforts and sales or possession of products. Independent research operators have worked out ingenious systems of sales measurement, even for single insertions of an advertising campaign. Daniel Starch, in particular, set up elaborate controls in an attempt to sift out exclusively advertising influence on sales[117]. Purchases of products by magazine readers - both before and after issuence - divided according to claims of reading or nonreading of brand advertisements were offered as evidence of sales effects. This approach logically represented an advance in control of related variables, but slow acceptance by the advertising industry may have been caused by the problem of obtaining accurate field data on advertisement reading and on dates of product purchases. Certainly the method has not been generally accepted as a sales evaluator of national advertisements in magazines.

Daniel Starch has presented additional evidence that readership of advertising is closely related to the attracting of prospective buyers[118]. He discusses the assertion that high readership ads tends to attract nonprospects for the products and services advertised. This is not so, he says, and he presents a good deal of evidence indicating that the more readers an advertisement has, the more the actual number of prospects. (Also, the more prospects, the greater likelihood there is of more readers)." The

data clearly show that as advertising (either for your own brand or for competition) is stepped up, reduced or stopped, there are marked effects on the number of buyers and current users"[119]. There is the likelihood of course, that advertising appropriations often are either increased or decreased largely in anticipation of what sales tends are expected to be.

In 1961 Starch added to his analysis on the correlation method of estimating the sales influence of advertising.[120] By comparing the findings of several of his approaches to this type of analysis, he was able to demonstrate a degree of consistency which he felt confirmed the relatively simple "Netapps" (NET Ad Produced Purchases) method. This requires magazine issue readers or television program viewers merely to report on whether the advertisements were seen and whether the product had been purchased within one week after noting.

If 15 per cent of 30 ad- noters bought the product, this would account for 4.5 purchases. If 10 / of 70 non-ad-noters bought the product, this would account for 7 purchases. The total number of reported purchases is known to be 4.5 + 7.0, or 11.5.

However, if the 30 ad-noters had not seen the ad, and if 10 per cent would have bought as did the non-ad-noters, this would account for 3 of the 4.5 purchases they did make. Thus, only 1.5 purchases seem direct outcomes of ad-noting or reading, which would be 1.5 out of a total of 11.5, or 13.04 per cent. In this

case, the advertising would be credited with 13.04 per cent of the reported purchases within the week after publishing or broadcasting the advertisement.

The present authors find insufficient consideration given by Starch to certain factors which he treats foo lightly or not at all. There is the possibility that noters of specific advertisements are generally better ad- noters and buyers of advertised products. In fact, the Advertising Research Foundation has demonstrated that prospective buyers were 50 per cent better noters of advertisements in LIFE magazine than the nonprospects were[121].

It is also possible that noting of advertisements makes it easier to remember and report products purchased or that purchase of a product makes it easier to remember noting the advertisement. Perhaps more important is the fact that Starch Readership Service data are not sufficiently accurate to justify such precise analysis. As with other correlation studies, there is always the possibility that other factors which are not isolated or identified may vitiate the entire approach.

The work of Harry Deane Wolfe of the University of Wisconsin in isolating the influence of advertising also deserves comment[122]. Wolfe discusses his explorations with this precautionary statement: "It is doubtful that complete advertising measurement is likely to be attained. There are two many variables, complicating factors,

and uncontrollable influences acting onthe mind of the consumer. But an "all or nothing" complex in research seems childish; a combination of 50 / fact and 50 / hunch seems a far better basis for decision than is 100 / hunch"[123].

His final estimate of how much of sales ads "produce" depends upon a comparison of latest brand usage among those who could recall that brand's advertising messages and those who could not. His correlation analysis indicated that slightly over 25 per cent of $ 15 million in sales bore a relationship to knowledge of present advertising. One of Wolfe's most interesting contributions is the summary of other factors which contributed to the total sales of $ 15 million :

1. Consumers like the product,
2. Consumers know it is a nationally advertised product,
3. Consumers know the brand name,
4. Consumers once knew the advertising,
5. Consumers can find the product in their favourite store,
6. Consumers took advantage of a special promotion such as mailed coupons, in-box coupons, off-price sales in store,
7. Consumers were influenced by store advertising and store display,
8. Consumers bought it on impulse,
9. Consumers bought it because of communal influence (or word of mouth) exerted by parents, relatives, friends, dealers, professional people (doctors, dentists), and others[124].

The correlation approach, with respect to these and other factors, may result in the assertion: "But all of these factors were the same for both the exposed and unexposed groups". And the response is :"If there are all of these and possibly more factors influencing the sale, then how can correlation evidence be more than a crude estimate of advertising's effect?" Wolfe has met this rebuttal by his statement that 50 per cent fact is better than no fact at all.

However, it needs to be pointed out that if people abandon judgement which is 55 per cent right in favour of leaning on research which is 50 per cent right (and this often does happen at one accuracy level or another), there has been a loss in decision making efficiency. Moreover, this is usually done with an overt acknowledgment of the research inaccuracies. The poor research is often set forth as being "at least as right as it is wrong". But the real difficulty is to try to balance the likely accuracy of the research with the likely validity of judgements made without the research.

In any case, it is more conservative to evaluate whole campaigns than to rate each advertising insertion-sales trends over a period of months or years may show a parallel with advertising expenditures or readership ratings[125]. When magazine readers are separated into those noting and those not noting particular brand advertisements, the differential may show a convincing pattern.

But there remain possible influences other than advertising. People who do not see the magazine advertisement of a particular brand, for example, must obviously include those who noted few advertisements of any kind. The people who read few advertisements may act differently as consumers in comparison with people inclined to read a great many advertisements. This does not deny the importance of the evidence, but raises a serious question about crediting all sales advantages to advertising alone.

A Direct-Mail method :

A great difficulty in evaluating advertising in terms of sales effectiveness has been that of finding an acceptable criterion. But the criterion might well be sales - if we can find a "pure" measure of sales directly related to advertising!

An ingenious way to control most of the extraneous variables which might influence sales (other than advertising) has been proposed by Dik Warren Twedt.

In a direct-mail test, evaluation of different appeals can be objective and immediate. If the subsamples are unbiased and if the differences in returns are significant, we can usually safely attribute obtained sales differences to the differences in the advertisements.

Accordingly, Twedt suggests that we develop 10 different advertising layouts for a product, for example. In each layout, the product and the price are exactly the same, but the layouts are

purposely constructed to maximize differences in responses, therefore, including some ads which we believe will "pull" rather poorly.

At this point we need a known, relatively homogeneous population for the test. Suppose that a department store in a large city has a mailing list of 45,000 charge customers. A mailing of each of the 10 forms of the advertisement could be made to every tenth name on the list; that is, each advertisement would be mailed to 4,500 potential buyers. The order would be :

Customer	Advertisement and possible appeal
1	A (quality)
2	B (protection)
3	C (ease of use)
4	D (curiosity)
5	E (service)
6	F (scientific)
7	G (announcement)
8	H (how this product came to be)
9	I (testimonial)
10	J (snob-appeal)

The cycle would then be repeated throughout the list.

The department store mailing would be in the form of an enclosure of a flyer with the monthly statement. Since a large department store would already have a regular schedule of statement

enclosures, the additional expenses would be in the preparation and production of 10 layouts instead of 1; the collating of outgoing envelopes so that bias in the 10 subsamples is minimized; and perhaps additional postage.

If each layout offered the same price inducement, and of a sizable sort, it seems reasonable to assume at least a 5 per cent return on the mailings. On this basis, the average return per layout would be 200 or more orders. Differences in return would provide helpful evidence as to the most promising appeal to be used for advertising in a national medium.

OTHER METHODS :-

Mathematical models for partial and multiple correlations of all factors that might influence sales may make a growing contribution to the evaluation of advertising. The first decade of extensive application of high-speed computers to assess the sales influence of advertising has made some research analysts optimistic[126]. Using controlled advertising situations in which intensity and type of promotion varied, the results of one study suggested that by use of a mathematical model the quantities needed to evaluate and compare alternative promotional compangns could be computed. But there are reservations and limitations.

E.I. du Pont de Nemours company has pioneered in the application of mathematical operations research models to advertising. Some of

the basic considerations are worth quoting :

"Simplified, the general idea is to identify and isolate all the separate factors involved in sales, then compare the contribution each makes in getting people to buy the product.

The first step, identification, is relatively simple. Most of the factors contributing to sales - weather, salesmen, dealers, buying power, to name just a few - are obvious and fairly common everywhere. Isolating and comparing the effects of each of the sales factors is far more difficult.

The patterns, and data that emerge from an analysis of the products previous sales are used to build a mathematical model. Once the mathematical model is built, the job of isolating and comparing all the factors affecting sales begins.

How is this done? Ideally, all factors except one are held constant. By doing this with every factor involved, we eventually get a pretty good picture of how each one affects or relates to the total sales problem under analysis.

Unfortunately .... some factors affecting selling, such as weather, population and the consumer's buying power, are completely beyond the advertiser's control. Others, the salesman's personality, for example, are generally not even ascertainable.

Can we manipulate the advertising factor to determine its effect on sales? With a product that has never, before been

advertised this is feasible .... a product that's been previously advertised obviously doesn't lend itself to this pat method of measurement.

As the next best thing, du Pont's advertising research section uses a technique called evolutionary operations. By varying the advertising expenditures year by year in some planned fashion .... and by equalizing out the other factors by controlling them or at least by measuring their variability, we isolate the advertising factor"[127].

Essentially, there are two ways of obtaining marketing data for mathematical evaluations of advertising expenditures. The practical, unrefined approach is to take the company records of sales and of all promotional expenditures and analyse their relationships. While this approach is centred upon wholly normal advertising conditions, it is beset with the many variables in promotional efforts and marketing conditions. As a measuring approach, it is also blurred by the fact that business management guides its promotional moves according to jugements of opportunity for gain.

The alternative to retrospective study of company marketing records is to permit some control of promotional efforts for making advertising research more definitive. Control may take such forms as holding other promotional efforts fairly constant while varying the application of advertising - or setting aside selected marketing areas or segments for experimental advertising study.

A more extreme concession to research purposes is to turn all promotional decisions over to the demands of experimental design for a period of exploration. While it is next to impossible to set up a controlled experiment which meets all practical requirements, it is even more difficult under normal operating conditions to abstract definitive evidence on the sales influence of most national advertising.

The limitations of correlated evidence of the sales influence of advertising are complicated. The normal promotional activities of a company are opportunitic, interrelated, and at times erratic. If a new merchandising man or advertising manager happens to come on the scene while data for advertising analysis are being collected, his impact may completely overshadow ordinary advertising changes. New personnel, new product models, changed consumer demand, and an endless array of other factors may make evaluation of advertising extremely difficult and tenuous.

Also, as soon as research is permitted to control promotional activities, other problems arise. Usually there is no such thing as holding other promotional activities constants, since they are not normally constant. If sales territories are assigned to research, there arises the problem of comparability between territories; and there is also the question as to whether knowledge learned presently in one territory will have valid application later in the whole market.

In other words, there still may be advantages in the partially

controlled market study, but the practical solutions and applications are decidedly limited.

The ideal research, based on correlation of variables, calls for complete experimental design in which all variables are accounted for or actually controlled. Such experiments, usually local, make it possible to study the effects of varied advertising and to compare different advertising copy and strategies in Delected markets. Some of the problems include variations in the operations of competitors, basic variations in test markets, changes related to time, and completely controlled promotion. Decisions must be objectively scheduled - never to play a haunch or to exploit an opportunity, as in normal business. The experimental approach is certainly the best way to obtain data on advertising influence, but the ideal advertising experiment has not yet been worked out.

Computers :-

Despite the limitations and precautions indicated for mathematical analysis of advertising effects, the development of electronic computers has brought a promising approach to advertising research. It would appear that as fast as man can evolve models or formulas for advertising analysis, computer technology will progress to make solutions possible and practicable. These twin techniques represent tools which may eventually enable research to move step by step toward more complete and more valid estimates of the sales influence of all marketing forces.

All mathematical explorations of relationships between national advertising and sales are subject to a number of constraints. Computers have mechanical precision, and it is usually possible to set up and solve appropriate mathematical models. Restraint begins with the fact that models require the correct selection of the right variables and the assignment of appropriate and proper weights.

Models also involve assumptions, some of which cannot be tested for their validity. Traditionally the data relating to advertising have lacked precision, and fine instruments or analysis do not improve the data.

For these reasons, our summary regarding possible mathematical analysis of the effects of advertising on sales response is as follows :

Levels of accuracy in advertising research are lower than in most other areas of application of these techniques.

The procedure does not correct for faulty or inadequate assumptions.

Weights and relationships included in design of the model are not validated or corrected.

Computations cannot overcome the limitations imposed by poor data. (A computer is not a 'brain' but a good 'right arm').

Deviations from normal advertising operations for the purpose of obtaining the data must be allowed for in applying the findings.

It is easier to assess the influence of gross advertising expenditure than the effects of campaigns or individual media; and it is easier to evaluate a campaign than to isolate individual advertisements or insertions.

Pretesting in selected sales areas :

Sales - area testing has long been a part of advertising research. Groups of cities and/or markets are selected, using some cities as controls without advertising changes. The sales pattern for the product is checked through store inventories for several weeks before the test. Then new advertisements are introduced, each variation being tried in several cities and checked for sales results in many stores. Inventories of stock are kept during the test period and may be extended some weeks thereafter to study the normal sales pattern. If certain advertisements are followed by sales increases well above the normal fluctuations in the same markets, they have an increased prospect of proving effective in later national coverage.

No two cities or markets are actually identical, and the selection of test cities is extremely important for sales area tests. Cities should be comparable in such population characteristics as economic levels, occupational distribution, national origins, and permanence of residence. Types of industry and everything related to the purchase pattern for the product should be matched. Naturally, the selection of cities should be aimed at

matching the characteristics of the total market in which the product is sold.

Dealers play a crucial role in the test, particularly if they keep the stock inventories necessary for measuring sales. Since all competing products should be checked, it is possible to keep the dealers from knowing the identity of the brand under test. Otherwise, they may be too cooperative and "push" the product.

Even if the investigator keeps track of inventory, which is the wiser procedure, he must keep account of all stock coming to the shelves from elsewhere within the stores, from warehouses, wholesalers, drop shippers, wagon distributors, the manufacturer, and even through loan from other sources.

Ready availability of stock at all times is also highly important. Two or more inventories before the test period, frequent inventories during the test, and two or more inventories following the test are recommended. The final inventories, especially in control cities, can help to ensure that the effects of the test advertisements are not confused with normal sales variations.

The timing of a sales-area test, the selection of media, the schedule of insertions, and the advertising plan must all be well coordinated. The test should be scheduled at such a time that the results will be most significant for a particular time at which the advertising is to be released generally. Buying trends for the product and its competition, as well as general business trends,

should be considered. The local media being used should relate as closely as possible to the national media for later use. The rate insertions should not be speeded up to a point which might distort the relative normal performances of different test advertisements. Even more important, the test copy should not be designed exclusively for immediate sales unless this is the strategy intended for the national campaign, and this last point clearly limits the kinds of advertising for which sales - area tests are adapted.

Sales-area tests always produce differences. The problem, then, is to evaluate the differences and to estimate the sales, effectiveness of each advertising design.

Early applications of this method led to unsupported conclusions, because normal sales variations by store and by community were not examined. There was little internal analysis of the data. After "winning" advertisements were selected and extended to national markets, there was often reason to believe that results were poor. Re-examination of the sales-area test data, in the majority of cases, showed that the differences obtained under test were smaller than the usual sales fluctuations in the same markets!

Sales-area tests should be used mostly for new products which are frequently purchased. If the product is purchased after long intervals or if the purchase pattern tends to be erratic, it is not practical to measure the effects of ordinary changes in advertising. Since sales-area tests necessarily cover brief periods, the results cannot be helpful in evaluating the campaign buildup and

cumulative effect typically sought through national advertising.

There has been a growing effort on the part of users of sales-area tests to extend their test areas to major regional markets and to seek fuller control of all promotional efforts in assigned regions. George H. Brown of the Ford Motor Company has given a broad description of such efforts in the evaluating of advertising copy. and media[128].

When an advertisement appears, it can do three things. It can make the name of a brand known, it can make this name known favourably, and it can cause buying; in short, it can cause brand awareness, brand preference, and brand purchase. How can we measure buying action? And how can we measure the net buying effort an advertisement may cause? Suppose we know how many readers of a publication buy a given brand. Then suppose an advertisement is inserted in this publication. Will any more readers of the publication buy the brand? If actually no more buy, do any buy who would otherwise not buy? That is the realistic measure of advertising effectiveness. Failure to increase volume of a brand does not mean that its advertising is ineffective. Ordinarily only new in the-market brands or brands with a minor share of the market are likely to be able to achieve net increases in share of volumes. Suppose, for example, that when there is no advertisement of brand in an issue, 15 / of the issue readers buy this brand, and whenthere is an advertisement in an issue, 17 / buy. If such a difference is demonstrated, it signifies that advertising causes buying.

Purchase Rates and ad-containing issues :

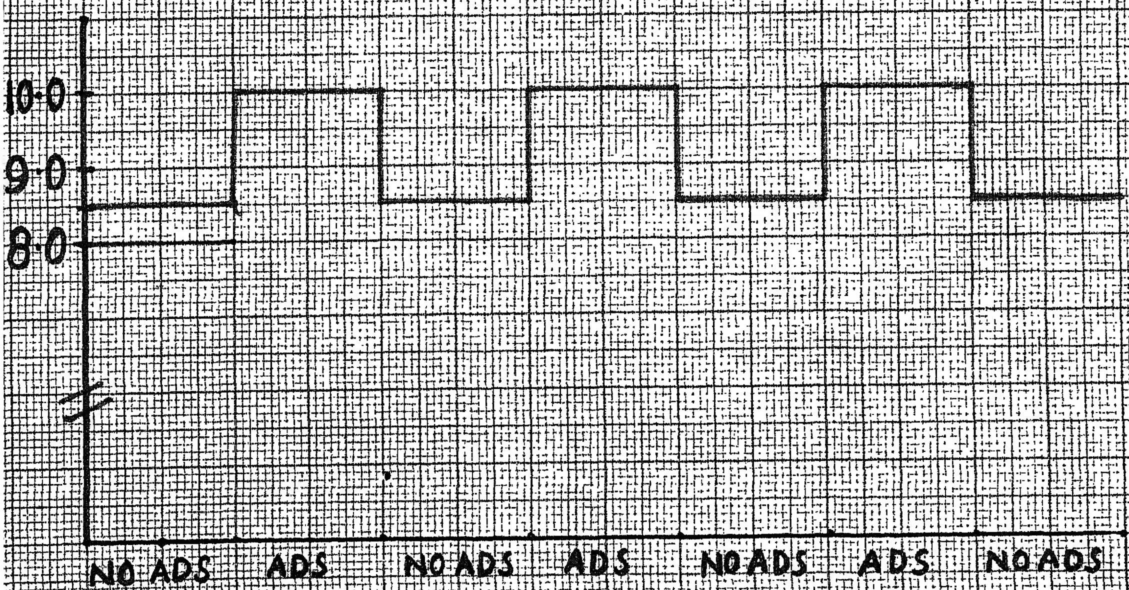
How do purchase rates differ when issues contain advertisements versus when they do not? Brand-purchase records of this type have been accumulated by Daniel Starch and Staff in connection with the long-range research project begun in 1944. This project has provided hundreds of instances of purchases of brands by issue readers the week following the distribution of issues that contained advertisements of the brands and the weeks following distribution of issues that did not contain advertisements. Take for example, the reader audience of the "Saturday Evening Post" for the years 1958 to 1963. The weeks when issues did not contain advertisements Life Saver Candies, 15.7 / of the readers bought Life Savers. When issues did contain advertisements, 20.3 / of the readers bought, a difference of 4.6 points in purchase rates. That is, 21 / more issue readers bought when issues contained Life Saver advertisements.

Another example - Kellogs' special K was a new cereal at the time. The weeks when issues of the Saturday Evening post did not contain advertisements of special K. 10.8 / of the women readers bought special K; when issues did contain advertisements, 12.8 / bought. The difference of 2.0 indicates that 18 / more women issue readers bought special K when issues contained advertisements.

Extensive purchase data for 73 brands, obtained for more than 110,000 audilt men and women, were compiled for the six years 1959 to 1964 when issues contained advertisements as well as when they

HIBIT – 1 –

PURCHASE RATE OF ISSUE READERS WHEN ISSUES CONTAINED ADVERTISEMENTS VERSUS WHEN THEY DID NOT.



did not. The number of advertisements for these brands was 707. They included food products, household supplies, drug products, candy, soft beverages, dentifrices, toilet soap, beer, cigarettes, and gasoline. 13.7 / of issue readers bought when issues contained advertisements of the brands. 11.55 / of issue readers bought when issues did not contain advertisements of the brands, 2.15 per centage points difference. 19 / more issue readers bought when issues contained advertisements of the brands[129]. The probability that a difference of 2.15 points would occur by chance is less than 1 in 100. The difference is therefore statistically significant.

Readership versus no-readership interviews:-

There were five brands - Budweiser, Clariol, Sprite, Tab, and Viceroy - for which there were advertisements in some issues and not in others. When issues did contain ads. 8.1 / of issue-readers bought. When issues did not contain ads. 6.9 / of issue readers bought. Additional purchases were 1.2 per 100 issue readers; i.e., when issues contained ads. 16 / more issue readers bought.

Exhibit 1 - Purchase rate of issue readers when issues contained advertisements versus when they did not.

Immediate and Delayed Effects of Advertising :

For convenience, effects of advertising may be classified as immediate and delayed. The immediate effects appear at once, i.e., chiefly within the first week or so after the appearance and perception of the advertising message. These effects consists of (1) increased purchases, particularly in the case of frequently purchased non durable goods, and (2) increased preference, particularly in the case of infrequently purchased durable products.

The delayed long-range effects consist of (1) brand loyalty, as indicated by purchases made without current advertisement stimulation in the case of frequently purchased products and (2) a reservoir of brand preference, as shown by the level of preference for a brand among non-ad readers.

The immediate effects are more striking, but the delayed effects are probably in the long run more profitable. The immediate effects represent return for advertising expenditure; the delayed effects, on the other hand, represent return on advertising outlay as a long-term investment.

The theory of advertising in the marketing process in a free-choice society consists in building a preferential associative link between a need and a brand name such that when the need arises a particular brand name will be so closely and favourably linked that it will induce buying action. This preferential link is built through promotional effort : (1) by inducing immediate buying

action and (2) by building and cementing the preferential link through brand use so that it will cause buying action when the need arises.

Corroboration by other studies : Studies made by the U.S. department of Agriculture (USDA), regarding the effect of advertising on the sale of frozen orange juice, lamb, and apples, showed increases of approximately the same magnitude. In these studies sales were checked in supermarkets during the weeks the product were advertised, and were compared with sales during the weeks they were not advertised. The findings differed for various products but showed increased sales of around 15 / during weeks with advertisements. Comparable findings are also reported by John B. Stewart of the Harvard Business School in a study on repetitive advertising in newspapers.

Starch has reported that obtaining purchase data by trailer questions after the completion of readership interviews but without reference to the advertisements does not lead to more ready recall of the brands purchased by ad readers than would otherwise be the case. This finding is corroborated by the USDA studies. In these researches there was no identification or interviewing of the readers of the advertisements, since effects were measured by sales in supermarkets. Nevertheless, increase in purchases due to advertising were fully high as in our procedure.

The Advertiser's Dollar (or Rupee) :

What does the advertiser get for his dollar (1) in immediate effects and (2) in delayed long-range effects? As stated previously, the delayed long-range effects, which consist of loyal repeat buyers, are probably the most valuable. Repeat buyers are built up by contimued advertising stimulation and by continued satisfactory use performance of the product.

First, let us see what the immediate effects are as measured in terms of net-ad-produced purchases (NETAPPS) per dollar of advertising cost. Let us picture the situation this way. An advertiser spends $ 44,250 for a one-page four-color ad for a candy bar in a publication of wide circulation. Furthermore, the advertiser inserts five or six such advertisements a year for a period of years. One may well ask, How can a $ 44,250 advertisement pay for a small 10-cent candy package? What do we find in the way of purchases of this brand, first, when there are advertisements in issues and, second, when they are not?.

How do these findings relate to advertising costs? To answer this question, we must ascertain the value of the purchases bought by the issue readers. This candy sells for 10 cents per package, but we find that consumers frequently buy more than one package at a time. In fact, purchases average about 14 cents. We further find that not only do more issue raders buy when issues carry advertisements of the brand, but those who read the advertisements make larger

purchases. Their purchases average 18 cents. This indicates, then, that the 6 / of additional ad-stimulated purchases among 11,000,000 issue readers make 660,000 additional purchases at an average cost of 18 cents, or a total of $ 118,000 of additional ad-stimulated purchases whenever an advertisement is carried in an issue. At a cost of $ 44,250 per advertisement it means additional purchases of $ 2.75 per dollar of advertising cost.

Another way of stating NETAPPS per dollar of advertising cost is in terms of purchases and costs per 100 issue readers. The Starch readership studies regularly report readers per dollar. In the case of this candy bar, the number of advertisement readers was 127 men and women ad readers per dollar of advertising space cost. This turns out to be an advertising cost of 39 cents per 100 issue readers.

The value of purchase per 100 issue readers is obtained this way. Since 6 points are the additional ad-stimulated purchases per 100 issue readers whenever an advertisement is carried, it means that, at 18 cents per purchase, the additional ad-stimulated purchases have a value of $ 1.08 per 100 issue readers. The $ 1.08 purchases and the 39 cent cost of advertising indicate $ 2.77 of ad-stimulated purchases per dollar of advertising cost, a figure substantially the same as the one shown by the preceding process of computation. The two are identical when computations are carried out several decimal places.

The advantage of the former procedure is that it makes use of the announced page rates (for example, $ 44,250), whereas the latter method makes use of the regularly reported ad-readers per dollar. The final figures are necessarily the same since they are derived from the same basic rate and readership data.

Exhibit 2 - Purchase of a Candy Bar when issues contain Advertisements versus when they do not

When there are advertisements in issues :	21 / of issue readers buy this brand.
When there are no advertisements in issues :	15 / of issue readers buy the brand.
Thus there is a difference of	6 / of additional issue readers who buy this brand when issues carry advertisements of the brand.

The long range effect built up by advertising and brand use consists of those users of the brand who currently buy from established preference. Data show that some 15 / of all issue readers buy without the immediate current advertising reminder. The repeat buyers represent a valuable asset built up by the advertiser and may be looked upon as an advertising investment rather than an advertising expenditure.

Net purchases and advertising cost : Data for many brands over a period of years indicate an average of about $ 3 of NETAPPS per $ 1 of advertising cost. This is probably a minimum figure, since effective advertising rates paid by large-volume users are no doubt less than the announced rates on which our readers - per dollar figures are necessarily based. To those executives accustomed to thinking of advertising expenditures in the neighbourhood of 3 % of total sales, this may appear to be a high cost for the additional return in sales. However, the usual 3 % figure is a percentage of all sales no matter how made, no matter what other cost items may enter, and no matter what share of the total sales would have been obtained anyway without the current advertising. If the 39 cents of advertising cost per 100 issue readers mentioned previously is related to total purchases (15) made per 100 issue readers in 10 weeks (5 advertisements per year), the 39 cents turns out to be about 2 % of total sales per 100 issue readers during that period of time. The USDA found in its study of the effect of advertising frozen orange juice that additional sales of about $ 4 were made per $ 1 of advertising cost. The New York brokerage firm Sharson, Hammill and Co. reported that they obtained about $ 3 of additional commissionable business per $ 1 of newspaper and broadcast advertising. The Oscar Meyer Co. similarly reported some $ 2.50 net additional sales of wieners of wholesale prices per $ 1 of radio advertising. At retail prices the ratio would probably be also around $ 3 to $ 1.

The NETAPPS method :-

The studies begun in 1944 and the continued accumulation of1 data by Daniel Starch and staff since then on the reading of advertisements and the concomitant use and purchase of brands led over these years to the development and verification of the NETAPPS method as a practical procedure for measuring the net purchases attributable to current advertisements. This method measures (1) net sales effects under normal day-to-day perception of advertising messages and (2) the normal day-to-day purchases of the brands advertised. There is no artificial setup involved. The method is designed to record and measure what people naturally do when they are normally exposed to advertisements and concurrently need to buy products. The procedure can be applied in any media markets where the following four kinds of data can be obtained :

Which persons and how many of them perceive - read, see, hear - a specific advertising message in a specific medium?

Which and how many of the perceivers ofthe specific advertising message buy the brand?

Which persons and how many of them do not perceive the specific advertising message?

Which and how many of the nonperceivers of the specific advertising message buy the brand?

The information is obtained from respondents by trailer questions after the readership interview has been completed, without

reference on the part of the interviewer to any advertisements or any relationship between ad-message perception and buying action. Whatever relationships exist are subsequently established from analysis of data. In the case of broadcast media it is necessary to ascertain not merely exposure to but actual perception of the advertising message, to obtain concurrent brand-purchase data, and then to analyse and measure relationships in terms of NETAPPS and dollar costs.

<u>The Unique features of the NETAPPS method</u> :-

1. It measures net advertising effects by comparing attitudes, behavior, and buying action of ad-exposed test groups (ad-perceivers) with non-ad-exposed control groups (non-perceives).

2. The test and control groups are closely comparable, probably more closely comparable than test and control groups set up in different areas or time periods can be, since they are both contemporaneous segments of the same media audiences.

3. It relates buying action directly to perception by identifying individual ad perceivers and brand buyers.

4. It is carried out currently as advertisements normally appear through wide-reaching media of communication, so that the respondent's ad perceptionand buying behavior occur in the normal day-to-day manner.

5. It does not require a costly experimental design which can be carried out only occasionally by larger companies.

6. It makes possible accumulation of data for individual advertisements which can be combined in such ways as may be desired for analysing and evaluating the effectiveness of compangins, copy themes, and forms of presentation.

7. Finally, it makes possible evaluation of dollar NETAPPS in relation to dollar advertising costs.

Recent approaches in measuring Advertising Effects :-

The various approaches used may be classified as inferential, quasi-direct, and direct.

The Inferential approach is illustrated by an analysis reported by Sim A. Kolliner, Jr., director of research of McGraw-Hill Publications, a division of McGraw-Hill, Inc. Kolliner refers to an analysis of sales and advertising costs of 99 companies during 1954 made by the National Industrial Conference Board. This study indicated that companies which put a larger share of their overall sales expense into advertising had a relatively lower sales expense in proportion to total sales. A similar analysis and similar findings based on 124 companies for the year 1960 were reported by American Supply and Machinery Manufacturers Association.

The Mc-Graw Hill study was designed to test this implied relationship between advertising and sales cost in more detail and

on a larger scale. The analysis was based on data for the year 1961 for 893 companies ranging in size from less than 1 million dollars to over 25 million dollars of annual sales. These 893 companies reported an average sales expense of 10 / of total sales of the 10 / sales expense, the share for advertising was 21.7 / . When the 893 companies were split into three groups according to the proportion of sales cost spent for advertising, the low advertising, group, whose advertising was 14.2 / of total sales expense, had an overall sales expense of 11.6 / . The middle income group whose advertising was 14.3 to 24.8 / of total sales expense, had an overall sales expense of 9.9 / , and the high-advertising group, whose advertising was over 24.9 / of sales expense, had an overall sales expense of 8.7 / . According to the report, the same relationship held for small companies as for large companies and for companies manufacturing different types of products. The implication is that the higher the percentage of advertising expenses is to total sales expense, the lower the percentage of total sales expense tends to be to total sales.

These are averages for many companies. Individual companies undoubtedly deviate widely and no doubt negatively in some instances. The findings can hardly be used to measure the effect of advertising for an individual company or product, or for evaluating the effectiveness of different compangns or advertising techniques and strategies. The compilation, however, tends to show the broad effect of advertising on sales.

The Quasi-direct approach :-

Experimental studies conducted by the marketing Research Division of the USDA, illustrate the quasi-direct approach[130]. One of these studies undertook to measure the effect of the advertising of lamb. The design of the experiment consisted of controlled amounts of promotion in six cities - three Northeast cities, Philadelphia, Syracuse and Springfield - Holyake, and three midwest cities, St Louis, Omaha and Des Moines. The tests were conducted in three 6-week periods between September 6, 1960 and February 11, 1961, with a gap over the holiday period.

The promotion was carried out in rotation in the three North-east cities and on a similar schedule in the three Midwest cities. There were two forms of promotion (1) the regular advertising by the American lamb council, and (2) the additional promotion by the cooperating retailers. Program A was carried on in one city, program B in the second city, and program C in the third city with no sponsored promotion, as a control. This schedule was rotated in each of the three groups of cities during the three 6-week test periods, so that each city had each of these three treatments.

Arrangements were made with 78 supermarkets in the six cities to measure sales of lamb by customary store inventory methods. The American Lamb Council's regular advertising consisted of approximately four newspaper - advertisements in each test city during the 6-week test periods. This was supported by cooperative advertising by the

retailers. Newspaper advertisements ranged from full-page four-colour spaces to two-colour three-quarter pages, and black-and-white two-third pages.

The experiment showed (1) that the American Lamb Councils' regular advertising program increased sales about 10 / over no promotion and (2) that the retailer cooperative promotion increased sales an additional 15 / .

A similar study was made by the Marketing Economics Division regarding the effect of advertising on the sales of frozen concentrated orange juice[131]. The findings indicated that monthly sales of concentrated orange juice during the 7-month active nationwide advertising compaign were 13 / higher than would be expected without promotional effort. Total advertising expenditures of about 4 million dollars produced an estimated sales revenue of about 18 million dollars. This indicates about $ 4.50 of additional sales per $ 1 of advertising costs. In another USDA study[132] it was estimated that $ 3,500,000 of advertising expenditure produced $ 13,300,000 of additional sales of frozen concentrated orange juice, or $ 3.80 per $ 1 of promotional cost.

The third USDA study[133] followed a similar plan and was designed to measure the effect of a special promotional program on the sale of apples. The outcome was that sales increased about 15 / (20 / when the use theme was stressed and 9 / when the health theme was emphasized.

The experiments were well-designed and came close to being direct measurements of the net effects of advertising. They are, however, not fully direct in the sense that they do not indicate which people and how many of them saw or read the advertisements, and which people and how many among them bought the various products. This further information is desirable in relating cause-and-effect more closely. These investigations are nevertheless important contributions to the study of so complex a field of inquiry.

The Harvard-Fort Wayne experiment. A different type of experimental study was carried out by John B. Stewart under the auspices ofthe Harvard Business school and reported in detail in "Repetitive Advertising in newspapers 1964". The initiative for this research came from the Newsprint Information Committee. The project was financed largely by this Committee and by the associates of the Harvard Business School. The purpose of the research was to test the effect of repetition of the same two unchanged 1,000 line ads in the introduction of two new products called Lestare Dry Bleach and Chicken Sara Lee. The experiment was well-designed and was carried out in Fort Wayne, Indiana. Exclusive of a small downtime business area, Fort Wayne was divided into four quadrants.

The No-exposure Area : Newspaper subscribers within the first quadrant received copies which did not contain advertisements for either product during the test-period. This section of the city was used as a control area for comparison with the other quadrants, which

were to receive different amounts of advertising exposure through the Fort Wayne News-sentinel.

The 4-exposure area : The newspapers delivered to subscribers in the second quadrant contained an advertisement for each product on the first four successive Wednesdays. After that, copies delivered in this area contained no advertisements for the test products.

The 8-exposure area : The newspapers delivered to subscribers in the third quadrant contained an advertisement of each test brand on the first eight successive Wednesdays and none after that time.

The 20-exposure area : The newspapers delivered to subscribers in the fourth quadrant contained an advertisement for each brand on twenty successive Wednesdays.

During the test period, interviews were made with 5,972 adult family members responsible for food shopping. Roughly about 1,500 interviews were conducted in each quadrant, at intervals throughout the test period ranging in number, during any one week from about 135 to as many as 357. The interviews in any one period were obtained between Thursday morning and the following Tuesday evening. No interviews were made on Wednesday, the day the advertisements appeared. The questionaire covered awareness of the advertising for the new brands, awareness of the names of the test products, information about the products, and trial purchases.

After the twentieth advertisement had appeared, 7.4 / of the respondents claimed to be aware of Lestare and 1.6 to have bought it,

20.1 / claimed to be aware of Chicken Sara Lee and 2.3 / to have bought it. The net effects of advertising on brand awareness or on trial purchases were determined by substracting the level of brand awareness or purchases by the control group from the level of awareness or purchases by the group exposed to the advertisements. The largest net Lestare awareness was 5.5 / in the sixth week, and the largest net awareness of Chicken Sara Lee was 22 / in the sixteenth week.

A substantial number of households purchased these new products during the 20-week test period. The following exhibit indicates purchases during the different exposure periods in the four areas, as estimated from the smoothed curves in Professor Stewart's book[134].

Exhibit - Cumulative Purchases by Newspapers Subscribers

		Percentage of households having purchased the brand at end of		
Brand	Area	4 weeks	8 weeks	20 weeks
Chicken Sara Lee	No - exposure	0.2	0.7	1.1
	4 - exposure	0.9	1.0	1.4
	8 - exposure		1.8	2.2
	20 - exposure			4.1
Lestare	No - exposure	0.6	1.2	3.2
	4 - exposure	0.8	1.0	1.8
	8 - exposure		1.3	1.0
	20 - exposure			2.8

The net effect of advertising after various exposures is indicated by subtracting the figure for the no-exposure area from the figures for the various exposure areas. The net number who bought Chicken Sara Lee after 20 exposures was 3.0 / (4.1 -1.1). After 8 exposures it was 1.1 / (1.8 - 0.7) and after 4 exposures it was 0.7 / (0.9 - 0.2). The number of purchases attributable to advertising was approximately proportional to the number of advertisement exposures. The number of advertisements in the 20-exposure area was five times the number in the 4-exposure area, and the net number of purchases was between four and five times as many (3.0 versus 0.7).

The Lestare figures are perplexing. The obvious inferences would be that purchases in the area which had 8 weekly advertisements had increased scarcely any more than in the no-exposure area and that after 20 weekly advertisements purchases had increased less than in the no-exposure area. The difficulty appears to be with the data for the no-exposure area. The no-exposure figures for Lestare are substantially higher than those for Chicken Sara Lee, particularly the jump from the 8-week figures to the 20 week no-exposure figure. For Chicken Sara Lee, the change in the no-exposure area was from 0.7 to 1.1 /, while for Lestare it was from 1.2 to 3.2 /. Apart from the no-exposure data, the figures for Lestare, however, are fairly consistent. The gross number of purchases after 20 exposures (2.8 /) was four to five times as many as after 4-exposures, just as the Chicken Sara Lee gross purchases after 20 exposures (4.1 /) were four to five times as many as after 4 exposures. Something unknown appears to have occurred in connection with Lestare purchases in

the no-exposure control area. Could it be that the retailers in this area did more than the retailers in the other areas to bring Lestare to the attention of customers? Were instructions to retailers and investigators followed meticulously?

<u>Limitations of the Methods</u>: These two types of experimental studies, i.e. the USDA studies and the Harvard-Fort Wayne study, measure net effects of advertising by using the technique of either control time periods or control areas with no advertising exposure in comparison with time periods or areas with different amounts of advertising exposure. Various factors, however, beyond the experimenter's supervision or anticipation may and usually do intrude, such as promotional activities by competitors, special pricing or couponing, weather conditions, andv various amounts of crossover of people and newspaper issues containing test advertisements between areas. And finally, there is the question of strict comparability between successive time periods or between test areas. In the Harvard-Fort Wayne study, the Chicken Sara Lee findings were consistent and decisive. The Lestare findings were partly so but turned up with the perplexing paradox of a larger number of purchasers (3.2 /) in the no-exposure control area than even in the 20 - exposure area (2.8 /).

The Harvard-Fort Wayne experiments, and the USDA studies were well-designed and are valuable in demonstrating that the net current effects of advertising are tangible and can be measured in terms of additional sales that would not be made except for the advertising.

The limitations of these methods are two. They do not relate directly, within the behaviour of the same persons, ad message perception and effects produced. They do not broadly, but nuvertheless indirectly, relate advertising and sales. By these indirect procedures, it is not readily possible to evaluate types of advertising themes and forms of presentation - the very elements which are the potent factors in advertising dynamics. In these kinds of tests it is not possible to separate the two essential components of effective advertisements, ability to achieve perceptions (i.e. readership) and ability to influence perceivers to buy (i.e., activating power). Marketing strategists are to-day keenly aware of the force of message factors, as witnessed by the recent emphasis on imaginative skill in creating advertising. But the fact remains that a powerful message can operate only on perceivers. The number of sales made by an advertisement, like the number of sales made by the circus pitchman, is just as dependent on the number of people induced to read or listen to the pitch as it is on the proportion of readers or listeners who buy. A further limitation is that, from the standpoint of the average company, such studies can be carried out only occasionally and at high cost.

<u>The Direct approach</u> :-

What is needed is a procedure whereby advertising performance and its net persuasive - activating effects can be continuously and currently evaluated through a direct approach, such as the NETAPPS procedure discussed in a preceding section. Other approaches to

measuring advertising effects, in this instance via broadcast media, are studie by James B. Landis on television advertising and a study by Thomas E. Coffin on intermedia effects.

Evaluating TV advertising effectiveness :-

James B. Landis designed a study to determine the net effect of TV advertising by attempting to evaluate various non-advertising factors in accounting for the difference in buying rate between viewer and nonviewers[135]. The study was based on 426 viewers (26.7 /) and 1,169 nonviewers (73.3 /). The buying rate of the advertised brand (not identified) among viewers was 28.4 / and among nonviewers 19.0 / , a difference of 9.4 points. Landis undertook to find out how much of this 9.4 difference might be due to non-advertising factors. He chose twelve factors, among them age, income, family size, and city size, and determined or estimated what part each had in the 9.4 difference. These amounts Landis emphasized, were largely estimates. However, he arrived at a total combined effect of these twelve factors of 3.0 points out of the 9.4 spread, leaving a net difference of 6.4 as due to the television advertising. The difficulties in this task are to make sure that all the important nonadvertising elements are included and, even more difficult, to obtain factual data for measuring precisely the magnitude of each factor. A net increase of 6.4 points in purchase rate over the non-viewers rate of 19.0 / would mean an increase of 34 / in buying rate due to advertising. This would be a large increase as compared with findings reported in various studies summarised previously. It

leaves open the question whether all other influential factors besides advertising have been found and whether the correct values have been assigned.

TV and Magazine intermedia advertising effects :-

Thomas E. Coffin under-took to measure intermedia effects of various combinations of advertising on TV network programs and a weekly magazine[136]. The study covered twenty-two brands advertised on the network program and in one weekly magazine. The respondents were 2,441 heads of households. Readers or viewers were those who reported having read one or more of the last four weekly issues or watched one or more of the last four broadcasts. The unit of exposure, therefore was the advertising vehicle rather than the number of commercials or advertisements. Brand purchases had taken place in the preceding four weeks. The interviews were conducted with the same respondents in two waves separated by three months.

Dr. Coffin found that the percentage buying the average brand among those not exposed to this advertising was 16.9 / ; among those exposed to advertising (TV and/or magazine) it was 20.5 / . He further found that the buying rate was related to the degree of exposure to advertising.

Data were reported for sixteen possible combinations of network and magazine in the two interview periods. The sample sizes for the individual groups were in some instances too small to yield significant figures. Furthermore, since the unit of exposure was

the medium, the figures are less meaningful than they would be if the unit of exposure were the number of advertising messages.

During a four-week period, TV advertising messages might be weekly or even daily, whereas seldom or almost never does a weekly magazine carry an advertisement for the same brand in every issue. It is desirable to relate advertising effects not only to media exposure but to the number of message exposures and especially to the number of message perceptions. Media and message exposures are not equivalent to message perceptions.

The Oscar Mayer Radio experiment :-

The Oscar Mayer Radio study was designed to ascertain the effect of radio advertising on brand awareness and purchase of wieners[137]. During the 1963 baseball season, Oscar Mayer sponsored 162 milwaukee Braves Radio broadcasts over 38 stations, at a cost of $ 80,000. Data regarding listening to games and purchase of wieners were obtained through 1,200 telephone interviews, 200 in each of the six Wisconsin cities during the last week of the broadcasts. Respondents were asked how many of the games they had heard, who sponsored the games, what brands of wieners they bought most often, and how many pounds of wieners they bought most often, and how many pounds of wieners they had used in the preceding thirty days.

Dr. Twedt reported that 42 / of the listeners to the milwauku Braves baseball radio broadcasts and 35 / of the nonlisteners bought wieners, of which 53 / were of the Oscar Mayer brand. There were

800,000 households in the area, of which 416,000 or 52 / , listened and 384,000, or 48 / , did not listen to these broadcasts.

The findings indicated net additional purchases, influenced by the advertising, of 401,000 pounds of Oscar Mayer Wieners over and above the poundage bought by nonlisteners, or \$ 200,000 of additional purchases at the wholesale price. Since the advertising cost was \$ 80,000, \$ 2,50 of additional sales were made per \$ 1 of advertising cost. This figure is in line with comparable figures of about \$ 3.00 at retail prices of net additional purchases per \$ 1 of advertising cost (average for many different products), found by the author and reported in a preceding chapter for a wide variety of products. This return may appear meager. However, it must be borne in mind that in the long run the chief value of purchases stimulated by advertising consists in building up a sizable backlog of loyal buyers who continue to make repeat purchases without current advertising stimulation. These repeat purchases constitute, in fact, for established widely distributed products, the largest and most valuable segment of total sales.

The NETAPPS procedure can be applied directly to the data. Since 53 / of the 42 / of listeners bought Oscar Mayer wieners, this meant that 22.26 / of the listeners bought the brand. Likewise, since 53 / of the 35 / nonlisteners bought this brand, this meant that 18.55 / of the nonlisteners bought. The difference in the purchase rate (22.26 - 18.55) was 3.71 / . This meant that 3.71 / of the 52 / of listeners, or 1.929 listeners per 100 households,

bought this brand of wieners after influence by the broadcasts. Since the listeners bought an average of 26 pounds of wieners, 0.50154 pounds net purchases per household were influenced by the radio broadcasts, or a total for the 800,000 households of 401,000 pounts, the same figure arrived at by Dr. Twedt.

In the Oscar Mayer experiment, the four necessary kinds of data were obtained, (1) the number of listeners, (2) the number of non-listeners, (3) the proportion of listeners who bought the brand, and (4) the proportion of nonlisteners who bought the brand. Net purchases induced by the advertising can then be determined by the NETAPPS procedure as outlined previously.

Measuring Effects of Advertisements of Infrequently purchased products:-

Since one-week purchase rates for infrequently purchased durables - automobiles, refrigerators and insurance, for example - are extremely low, usually a very small fraction of 1 / , such large samples are needed to achieve statistically significant differences in measurements, that the procedure of NETAPPS requires adaptation.

Brand awareness, acceptance, and purchases : The ultimate purpose of advertising is to sell the product. To this end, advertising is designed to create awareness of a brand, to forge a link between a need or want and a brand name, to create acceptance of the brand, and finally, to induce purchase. The advertisement emphasises the need and connects the need to a specific brand name. The sequence may be from name to need, or from need to name. The

advertisement may tend to connect a need to a brand name. Or, if the current need is not felt, the advertisement will tend to tie a brand name to an eventual need, say, for automobile transportation the connection is a two-way channel, and no matter which way the sequence via advertisements occurs, the occurrence of the linking process tends to strengthen the connection between a brand and the product when needed. Furthermore, perception of the message may not only cement the connection between name and need but modify the need. For example, a household has had the same electric refrigerator for eighteen years. It has been highly satisfactory, has required little servicing, and works as well as ever. Current advertisements, however, tell about new features. Awareness of these features tends to feed back to change the need, thus actually, creating a need that did not exist in the ad perceivers' mind before.

A similar two-way process goes on between ad-message perception and brand purchase and use. Ad-message perception tends to increase brand purchase and use, and in turn brand purchase and use tend to increase ad-message perception. Cognizance of these two-way sequences is particularly important in understanding and measuring the performance of advertisements of infrequently purchased durable products.

Readers show that readers exhibit a higher degree of brand acceptance, brand usage, and brand purchase than non-ad readers. The purchase rate (within the past year) of automobiles is about 75 / higher among ad readers than among on-ad readers. The purchase rate of refrigerators, washer dryers, and insurance is about twice as high

among ad readers as among non-ad readers. Not all of this higher purchase rate is due to the current advertisements, but a substantial, part of it is.

Let us see how this works in the case of automobile advertisements. Purchase of a make is the definite final measure of advertising performance, and usually serves as an adequate single measure of advertising effectiveness of frequently purchased products. Purchases of durables, however, are made relatively infrequently. This makes the measurement of brand awareness and brand acceptance, as preconditions to buying, relatively more important for durables than for nondurables. Frequently purchased low-cost products are commonly bought on a "trial" basis. They cost so little that they can be tried without either emotional commitment to the brand or expenditure of an important sum of money. Accordingly, brand acceptance is not a prerequisite to purchase and often is achieved only after use has established the products desirability. However, for durable products, particularly for expenseive durables, brand acceptance is usually a prerequisite for purchase. This is because people know they will not soon be able to change their decision. Their money is spent and they must make do for a long time with the product purchased. Much more caution in making the purchase is obviously indicated. Hence, brand acceptance is much more likely to precede purchase. For expensive durables, brand acceptance correlates very closely with actual brand purchase.

The following analysis is based on data obtained from men ad

readers and non-ad readers of (1) 420 advertisements in the Saturday Evening Post for 12 established makes of automobiles during the four years 1959 to 1962, (2) 109 advertisements for 6 recent compact models, and (3) 32 advertisements for two non-American makes. The 12 established makes were Buick, Cadillac, Chevrolet, Chrysler, Dodge, Ford, Imperial, Mercury, Oldsmobile, Plymouth, Pontiac, and Rambler. The 6 recent compact models were Comet, Corvair, Dart, Falcon, Tempest, and Valiant. The two non-American makes were Renault and Volksevagen.

Brand awareness of the 12 established makes was 100 / among both ad readers and non-ad readers. Consequently, awareness of makes no longer serves as a measure of advertising performance. Awareness of the 6 recent makes was 100 / among ad readers and 98 / among non ad readers. Awareness of the 2 foreign makes was 100 / among ad readers and 99 / among non-ad readers.

Brand acceptance is measured by asking respondents, If you needed an automobile, would you buy [name]? Brand acceptance of the 12 established makes was 19 / higher among ad readers (44.4 /) than among non-ad readers (37.4 /). Acceptance of the 6 recent models was 29 / higher among ad-readers (36.3 /) than among non-ad readers (28.2 /). Acceptance of the two non-American makes was 38 / higher among ad readers (30.7 /) than among non-ad readers (22.3 /).

Use or ownership of a brand is also a significant indicator of brand acceptance. Ownership of the 12 established makes was 39 / higher among ad readers (9.7 /) than among non-ad readers (7.0 /).

Purchase rate of the 12 established makes during the preceding

twelve months was 53 / higher among ad readers (2.09 /) than among non-ad readers (1.37 /). Purchases of the 6 recent makes were at a rate nearly three times as high among ad readers (0.91 /) as among non-ad readers (0.34 /). Figures are averages per make. Note that the widest relative difference between ad readers and non-ad readers was in their purchase rates. The data are summarised below :-

Automobiles

	Among non-ad readers, /	Among ad readers, /	/ higher among ad readers
Brand acceptance:			
12 established makes	37.4	44.4	19
6 recent makes	28.2	36.3	29
2 foreign makes (non U.S.)	22.3	30.7	38
Average, 20 makes	33.1	40.6	23
Brand use :			
12 established makes	7.0	9.7	39
6 recent makes	1.1	2.0	82
2 foreign makes (non U.S.)	1.2	2.9	142
Average, 20 makes	4.7	6.7	43
Brand purchase rates, yearly :			
12 established makes	1.37	2.09	53
6 recent makes	0.34	0.91	168
2 foreign makes	0.35	0.79	126
Average, 20 makes	0.96	1.61	68

Brand acceptance or favourable buying attitude correlates very closely with purchases. The higher the brand acceptance, the higher the purchases. The statistical correlations between brand acceptance and purchase, between acceptance and use, and between use and purchase were between 0.85 and 0.90.

Advertising plays a substantial part in creating the acceptance of the various makes and in stimulating long-term buying action. However, not all the wide difference in acceptance or long-term purchase between ad readers and non ad-readers was due to the reading of the current advertisements. When that is done, approximately the same relative amount of net additional acceptance is found to be due to current advertisements, as was the case with regard to net additional purchases of nondurable products attributable to the current ad reading. About a third to a half of the spread in acceptance level between ad readers and non-ad readers was attributable to current ad stimulation.

The message is the motive force of advertising. What it says and how it says it make the difference, and a very large one The unique advertisements of Volkswagen have been twice as effective in creating acceptance and inducing purchases as the general run of automobile advertising, 16 / as against 9 / . In terms of cost, the difference is even larger, 67 men readers per dollar as against 53.

Refrigerator Advertisements :

Nine makes of electric refrigerators had 83 advertisements during the four years 1959 to 1962. The brands are Admiral, Frigidaire, General Electric, Hot Point, Kelvinator, Philco, RCA Whirlpool, Westinghouse, and Wizard.

Brand-name awareness was 99 / among ad readers and 94 / among non-ad readers. A recent make, Wizard, had an awareness level of only 64 / among non-ad readers, but 90 / among ad readers. Brand acceptance was 26 / higher among ad readers (44.2 /) than among non-ad readers (35 /).

Purchases were at a rate just twice as high among ad readers (1.71 /) as among non-ad readers (0.86 /). Brand acceptance correlates very closely with actual purchases. The brand highest in acceptance was also the brand highest in rate of purchase. The lowest in acceptance was lowest in purchases. The other seven makes had almost identical corresponding ranks in acceptance level and purchase rate between the highest and the lowest. Statistically, the correlation was 92 out of 100.

The net additional brand acceptance, as shown by our formula, attributable to the current advertisements was approximately the same relative amount as was found in the case of other products. The same is true of purchases. Reading advertisements does create additional measurable brand acceptance, and does stimulate additional measurable brand acceptance, and does stimulate additional

measurable purchases. When there is no advertisement of a brand in an issue, brand acceptance among readers is the same as among non-ad readers, when there is an advertisement in the issue. Specifically, brand acceptance of these nine makes of refrigerators among issue readers was 36 / when there was no advertisement in the issue and 35 / among non-ad readers when there was an advertisement in the issue. Among ad readers, brand acceptance was 44.2 / .

Similarly, purchase rate of the nine makes by readers of non-ad-carrying issues was 0.78 / , by non-ad readers of ad-carrying issues, 0.86 / , and by ad readers, 1.71 / . These measurable differences are directly tied to ad-message perception.

To summarise, a basic function of advertising is to link consumer needs and product names so that when a need arises a brand name will favourably come to mind and lead to buying action. Effects of advertisments of infrequently purchased durable products can be measured by the same procedure as effects of advertisements of frequently purchased nondurable products. Brand awareness, acceptance, and purchase of infrequently purchased durable products are substantially higher among ad readers than among non-ad readers. Net ad-produced increases in brand awareness, acceptance and purchase are of approximately the same magnitude for infrequently purchased durable products as for frequently purchased nondurable products. Advertising appears to be as effective per dollar spent in selling infrequently purchased durable products as in selling frequently

purchased nondurable products.

The material obtained was from an all-India manufacturer of torches and cells. The firm - Geep Flashlight Industries Ltd., Allahabad, are the second largest manufacturers in this field. The material was painstakingly collected by the Sales and Advertising departments of Geep Flashlight, and checked by the author.

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PART IV

O B S E R V A T I O N S

O B S E R V A T I O N S

The following is the gist of the observations made on the basis of material obtained from Geep Flashlight Industries, Limited, Allahabad: "Combined" includes miscellaneous articles.

The figures, when lumped together into four quarters (each quarter being of three months), and arranged under the heads: Torches, Cells and Combined, assume the following form :-

PRODUCTWISE / QUARTERWISE ADVERTISING EXPENSES

ALL-INDIA FIGURES — 1972-73 (MAY 72 TO APRIL 73)

MEDIA BREAKUP	Ist QUARTER			IInd QUARTER		
	TORCH	CELL	COMBINED (OTHERS)	TORCH	CELL	COMBINED (OTHERS)
1. Press Advertising	0.750	0.750	0.120	0.750	0.750	0.200
2. Radio Advertising	-	-	1.800	-	.126	1.800
3. P.O.S. Material	.624	.628	-	1.067	.228	.245
4. Gift Articles	-	-	-	-	-	-
5. Display Scheme	-	-	-	-	-	.837
6. Outdoor Advertising	-	-	2.150	-	-	1.200
7. Match-box Advertising	-	-	.100	-	-	.108
8. Mailing	-	.120	-	-	-	.020
9. Vans	-	-	.088	-	-	.100
10. Fairs and Exhibition	-	-	-	-	-	.600
11. Film Advertising	-	-	-	-	-	-
12. General Reserve	-	-	.010	-	-	.010
TOTAL	1.374	1.498	4.268	1.817	1.104	5.120

ALL FIGURES IN Rs. LACS

IIIrd QUARTER			IVth QUARTER			TOTAL			GRAND TOTAL
TORCH	CELL	COMBINED (OTHERS)	TORCH	CELL	COMBINED (OTHERS)	TORCH	CELL	COMBINED (OTHERS)	
1.000	1.500	0.080	1.000	1.500	0.080	3.500	4.500	0.480	8.480
-	.540	1.800	-	.120	1.200	-	.786	6.600	7.386
-	.099	-	-	-	.825	1.691	.955	1.070	3.716
-	-	-	-	-	2.218	-	-	2.218	2.218
-	-	1.250	-	-	.755	-	-	2.842	2.842
-	-	1.600	-	-	2.750	-	-	7.700	7.700
-	-	-	-	-	.380	-	-	.588	.588
.070	-	.010	.020	-	.084	.090	.120	.114	.324
-	-	.120	-	-	.140	-	-	.448	.448
-	-	3.300	-	-	.238	-	-	4.138	4.138
-	-	1.470	-	-	1.366	-	-	2.836	2.836
-	-	.110	-	-	.282	-	-	.412	.412
1.070	2.139	9.740	1.020	1.620	10.318	5.281	6.361	29.446	41.088

MONTHWISE PRODUCTWISE SALE 1972-73 AND 1973-74 IN Rs.LACS - ALL INDIA FIGURES

MONTH	1972 - 73				1973 - 74			
	TORCHES	CELLS	BULBS	TOTAL	TORCHES	CELLS	BULBS	TOTAL
MAY	20.39	59.56	2.33	82.28	14.51	51.80	1.72	68.03
JUNE	14.07	63.00	1.84	78.91	22.90	53.47	1.92	78.29
JULY	21.68	59.49	1.42	82.59	19.83	51.46	0.90	72.24
AUGUST	18.76	60.46	1.47	80.69	21.32	59.14	1.45	81.91
SEPTEMBER	15.74	71.73	0.71	88.18	19.17	63.23	0.98	83.33
OCTOBER	8.75	59.33	0.81	68.89	15.12	59.71	0.83	75.66
NOVEMBER	9.58	59.74	1.81	71.13	19.53	87.95	1.05	108.53
DECEMBER	14.19	79.14	1.80	95.13	26.51	97.75	1.81	126.05
JANUARY	19.32	90.93	1.57	111.82	20.28	87.09	1.69	109.06
FEBRUARY	18.46	72.95	0.97	92.38	20.79	67.50	1.60	89.89
MARCH	6.63	54.18	0.83	61.64	24.17	111.50	1.74	137.41
APRIL	17.41	71.19	1.38	89.98	35.75	103.15	1.86	140.76
TOTAL	184.98	801.70	16.94	1003.62	259.73	893.73	17.50	1170.96
GOVT. COOP. AND CSD	40.61	10.09	0.01	50.71	0.62	1.59	-	2.21
EXPORT	2.90	-	-	2.90	8.38	1.43	-	9.81
TOTAL	228.49	811.79	16.95	1057.23	168.73	896.75	17.50	1182.98

PRODUCTWISE / QUARTERLY SALE 1972-73 AND 1973-74 IN Rs.LACS

QUARTER	1972-73				1973-74			
	TORCHES	CELLS	COMBINED (OTHERS)	TOTAL	TORCHES	CELLS	COMBINED (OTHERS)	TOTAL
FIRST QUARTER	56.14	182.05	5.59	243.78	57.29	156.73	4.54	218.56
SECOND QUARTER	43.25	191.52	2.99	237.76	55.61	182.08	3.26	240.95
THIRD QUARTER	43.09	229.81	5.18	278.08	66.32	272.79	4.55	343.66
FOURTH QUARTER	42.50	198.32	3.18	244.00	80.71	282.15	5.20	368.06
TOTAL	184.98	801.70	16.94	1003.62	259.93	893.75	17.55	1171.23
GOVT.,COOP. AND CSD	40.61	10.09	0.01	50.71	0.62	1.59	-	2.21
EXPORT	2.90	-	-	2.90	8.38	1.43	-	9.81
TOTAL	228.49	811.79	16.95	1057.23	268.93	896.77	17.55	1183.25

The following is the gist of the observations made on the basis of materials obtained from Pure Drinks Ltd., (manufacturers and marketers of the soft-drinks "Coca Cola" and "Fanta"), New Delhi.

Advertisement Expenses 1971-72

Month	Amount (in Rs.)
Nov. 71	26,942.45
Dec. 71	132.791.48
Jan. 72	222,957.47
Feb. 72	117,811.12
March 72	423,074.28
April 72	46,372.43
May 72	265,349.81
June 72	296,294.40
July 72	50,877.69
Aug. 72	229,091.29
Sept. 72	143,691.85
Oct. 72	453,168.03
TOTAL	2,408,512.30

Advertisement Expenses 1972-73

Month	Amount (in Rs.)
Nov. 72	50,808.46
Dec. 72	162,496.70
Jan. 73	313,221.72
Feb. 73	149,487.94
March 73	95,479.97
April 73	113,092.12
May 73	88,704.36
June 73	25,510.23
July 73	111,968.24
Aug. 73	98,400.46
Sept. 73	34,963.80
Oct. 73	691,640.00
TOTAL	1,935,774.00

Advertisement Expenses 1973-74

Month	Amount (in Rs.)
Nov. 73	25,976.90
Dec. 73	66,574.75
Jan. 74	93,873.66
Feb. 74	36,448.59
March 74	187,855.77
April 74	111,917.22
May 74	96,601.02
June 74	92,771.37
July 74	67,723.60
Aug. 74	120,526.74
Sept. 74	66,488.80
Oct. 74	701.163.65
TOTAL	1,667,922.07

Advertisement Expenses 1974-75

Month	Amount (in Rs.)
Nov. 74	40,056.40
Dec. 74	24,868.66
Jan. 75	57,551.27
Feb. 75	34,111.15
March 75	52,371.51
April 75	62,281.27
May 75	41,485.91
June 75	103,633.23
July 75	33,073.58
Aug. 75	68,874.79
Sept. 75	35,970.11
Oct. 75	394,205.77
TOTAL	928,483.65

1971-72 Sales Revenue-Pure Drinks Ltd., New Delhi (figures in Rs.)

Month	COCA - COLA	FANTA	TOTAL
Nov. 71	515,130	209,835	724,965
Dec. 71	259,320	77,848	337,168
Jan. 72	295,380	81,059	376,439
Feb. 72	390,473	115,504	505,977
March 72	1,231,543	457,447	1,688,990
April 72	1,516,143	703,899	2,220.042
May 72	1,980,044	1,110,041	3,090,085
June 72	1,490,862	982,158	2,473,020
July 72	1,442,158	833,351	2,275,509
Aug. 72	1,117,766	663,864	1,781,630
Sept. 72	1,079,765	645,430	1,725,195
Oct. 72	925,530	526,622	1,452,152
TOTAL	12,284,114	6,407,158	18,691,272

SALES REVENUE (IN Rs.) 1972 - 73

MONTH	COCA-COLA	FANTA	TOTAL
Nov. 72	666,258	337,344	1,003,602
Dec. 72	359,560	147,523	507,083
Jan. 73	265,419	78,250	343,669
Feb. 73	606,101	229,919	836,020
March 73	951,726	421,848	1,373,574
April 73	1,124,689	578,873	1,703,562
May 73	884,493	455,229	1,339,722
June 73	381,551	166,009	547,560
July 73	1,101,676	306,255	2,407,931
Aug. 73	998 362	383,793	1,382,155
Sept. 73	1,163,696	442,412	1,606,108
Oct. 73	1,007,334	383,869	1,391,203
TOTAL	10,510,865	3,934,324	14,445,189

SALES REVENUE (IN Rs.) 1973 - 74

MONTH	COCA-COLA	TANTA	TOTAL
Nov. 73	520,610	155,056	675,666
Dec. 73	248,670	51,240	299,910
Jan. 74	326,736	65,270	392,006
Feb. 74	550,817	137,886	688,703
March 74	1,118,985	327,290	1,446,275
April 74	1,653,049	593,559	2,246,608
May 74	1,554,127	622,997	2,177,124
June 74	1,406,379	546,575	1,952,954
July 74	1,130,817	442,670	1,573,487
Aug. 74	1,166,528	437,105	1,603,633
Sept. 74	1,247,308	463,858	1,711,166
Oct. 74	867,279	271,075	1,138,354
TOTAL	11,771,323	4,114,581	15,885,904

SALES REVENUE (IN Rs.) 1974-75

MONTH	COCA-COLA	FANTA	TOTAL
Nov. 74	472,052	116,664	588,716
Dec. 74	255,902	47,243	303,145
Jan. 75	215,392	36,531	251,923
Feb. 75	385,473	86,113	471,586
March 75	849,525	222,974	1,072,499
April 75	1,483,931	482,422	1,966,353
May 75	1,652,375	623,065	2,275,440
June 75	1,241,222	429,129	1,670,351
July 75	1,075,639	354,491	1,430,130
Aug. 75	1,009,338	307,234	1,316,572
Sept. 75	770,914	218,815	989,729
Oct. 75	1,078,952	330,377	1,409,329
TOTAL	10,490,715	3,255,158	13,745,873

The following is the gist of observations made on the basis of data obtained from the Delhi Cloth and General Mills Co. Ltd., (D.C.M.)

Year	Total Advertising Expenditure (Rs.)	
1971-72	1,16,57,264	(From P/L Accounts)
1972-73	99,41,415	(From P/L Accounts)
1973-74	1,38,25,264	(From P/L Accounts)

D.C.M.

Year	Total Sales Revenue (Rs.)	
1971-72	1,35,43,85,215	(From P/L Accounts)
1972-73	1,30,45,74,042	(From P/L Accounts)
1973-74	1,37,79,37,931	(From P/L Accounts)

(Note: D.C.M. is engaged in the production and marketing of Textiles, Sugar, Confectionary, Chemicals, Alcohol, Vanaspati, Vinyl, Rayon, Fertilizers and Electronic goods throughout, India).

PART V

D I S C U S S I O N S

PART I - DISCUSSIONS REGARDING COCA-COLA DATA

STEP 1 - Construction of Scatter Diagram

The scatter diagram can be constructed from the following data : (Advertising on, and sales of, Coca-Cola and Fanta).

Advertising (x) (in Rupees)	Sales (y) (in Rupees)
26,942.45	724,965
132,791.48	337,168
222,957.47	376,439
117,811.12	505,977
423,074.28	1,688,990
46,372.43	2,220,042
265,349.81	3,090,085
296,349.81	2,473,020
50,877.69	2,275,509
229,091.29	1,781,630
143,691.85	1,725,195
453,168.03	1,452,152
50,808.46	1,003,602
162,496.70	507,083
313,221.72	343,669
149,487.94	836,020
95,479.97	1,373,574
113,092.12	1,703,562
88,704.36	1,339,722
25,510.23	547,560
111,968.24	2,407,931

Advertising (x) (in Rupees)	Sales (y) (in Rupees)
98,400.46	1,382,155
34,963.80	1,606,108
691,640.00	1,391,203
25,976.90	675,666
66,574.75	299,910
93,873.66	392,006
36,448.59	688,703
187,855.77	1,446,275
111,917.22	2,246,608
96,601.02	2,177,124
92,771.37	1,952,954
67,723.60	1,573,487
120,526.74	1,603,633
66,488.80	1,711,166
701,163.65	1,138,354
40,056.40	588,716
24,868.66	303,145
57,551.27	251.923
34,111.15	471.586
52,371.51	1,072,499
62,281.27	1,966,353
41,485.91	2,275,440
103,633.23	1,670,351
33,073.58	1,430,130
68,874.79	1,316,572
35,970.00	989,729
394,205.77	1,409,329

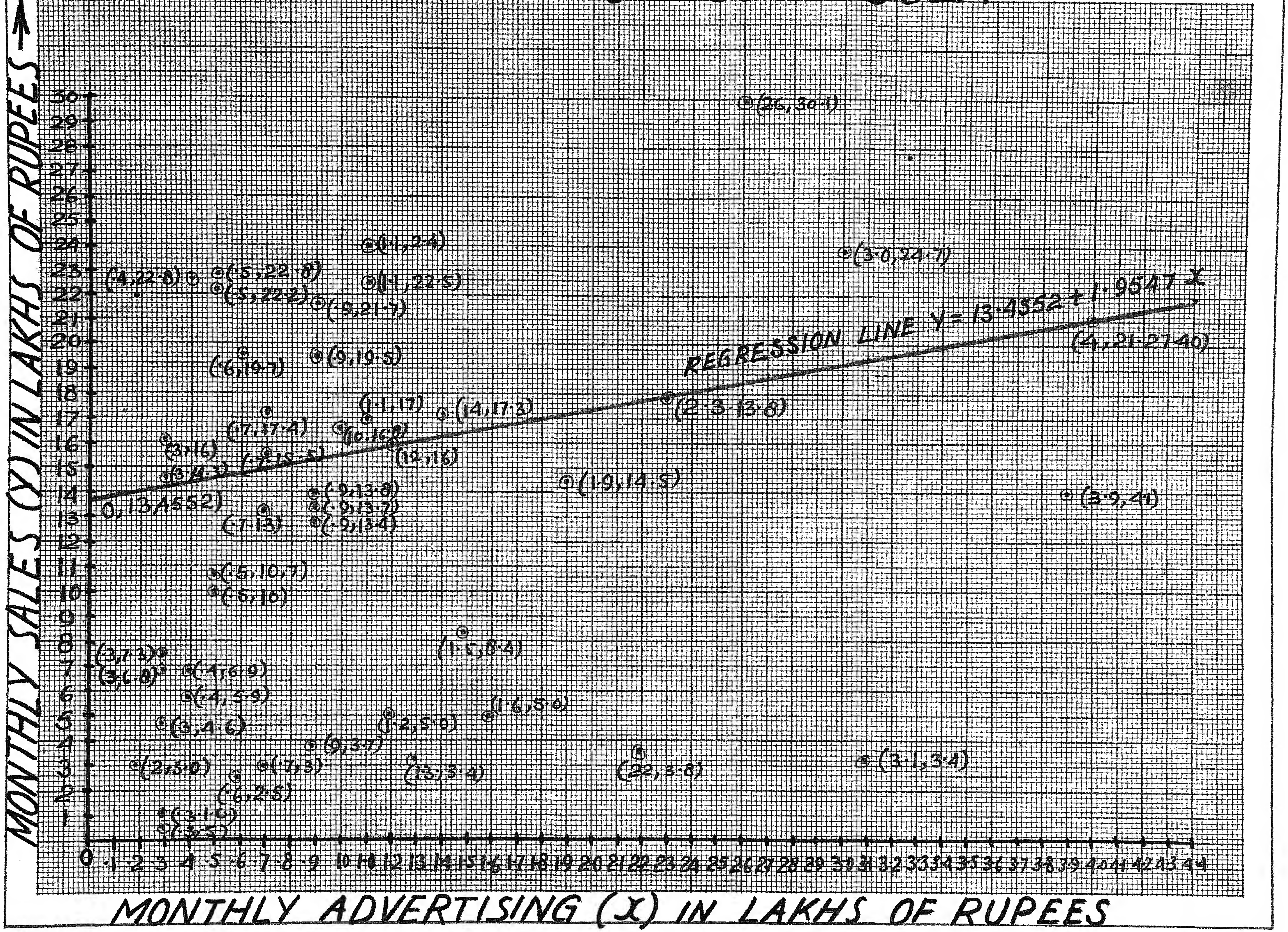
(It is obvious that towards the end of the year, a disproportionately high advertising expenditure has been incurred. This has

arisen from the "Treasury approach", spending the remaining portion of the advertising budget) (These end-of-the year figures should be ignored).

The figures on Advertising Expenditure and sales Revenue should be expressed tothe nearest lakh.

Advertising (x) (in lakhs of Rupees)	Sales (y) (in lakhs of Rupees)
0.3	7.3
2.2	3.4
1.2	5.0
0.5	22.2
2.6	30.1
3.0	24.7
0.5	22.8
2.3	17.8
1.4	17.3
0.5	10.0
1.6	5.0
3.1	3.4
1.5	8.4
0.9	13.7
1.1	17.0
0.9	13.4
0.3	0.5
1.1	24.0
0.9	13.8
0.3	16.0
0.3	6.8
0.7	3.0
0.9	3.9

SCATTER DIAGRAM FOR COCA-COLA



Advertising (x) (in lakhs of Rupees)	Sales (y) (in lakhs of Rupees)
0.4	6.9
1.9	14.5
1.1	22.5
0.9	21.7
0.9	19.5
0.7	15.7
1.2	16.0
0.7	17.1
0.4	5.9
0.2	3.0
0.6	2.5
0.3	4.7
0.5	10.7
0.6	19.7
0.4	22.7
1.0	16.7
0.3	14.3
0.7	13.1
0.3	1.0
3.9	14.1

SCATTER DIAGRAM FOR COCA-COLA

Observing the scatter diagram, we can obtain some distinct impressions. First, there is a clear positive relationship between advertising and sales; that is, sales, on the average, seem to increase with advertising. Second, the points seem to scatter from each other to a great extent, but they seem to have a (linear ?) relationship among them. Let us see whether linear regression analysis can be fruitfully applied to the sample data on hand.

STEP 2 - Linear Regression Analysis (for predictive equation)

Sums required for Regression Computations.

x	y	xy	x^2	y^2	$(x+y)$	$(x+y)^2$
0.3	7.3	2.19	0.09	53.29	7.6	57.76
1.3	3.4	4.42	1.69	11.56	4.7	22.09
2.2	3.8	8.36	4.84	14.44	6.0	36.00
1.2	5.0	6.00	1.44	25.00	6.2	38.44
0.5	22.2	11.10	0.25	492.84	22.7	515.29
2.6	30.1	78.26	6.76	906.01	32.7	1069.29
3.0	24.7	74.10	9.00	610.09	27.7	767.29
0.5	22.8	11.40	0.25	519.84	23.3	542.89
2.3	17.8	40.94	5.29	316.84	20.1	404.01
1.4	17.3	24.22	1.96	299.29	18.7	349.69
0.5	10.0	5.00	0.25	100.00	10.5	110.25
1.6	5.0	8.00	2.56	25.00	6.6	43.56
3.1	3.4	10.54	9.61	11.56	6.5	42.25
1.5	8.4	12.60	2.25	70.56	9.9	98.01

x	y	xy	x^2	y^2	(x+y)	$(x+y)^2$
0.9	13.7	12.33	0.81	187.69	14.6	213.16
1.1	17.0	18.70	1.21	289.00	18.1	327.61
0.9	13.4	12.06	0.81	179.56	14.3	204.49
0.3	0.5	0.15	0.09	0.25	0.8	0.64
1.1	24.0	26.40	1.21	576.00	25.1	630.01
0.9	13.8	12.42	0.81	190.44	14.7	216.09
0.	16.0	4.80	0.09	256.00	16.3	265.69
0.3	6.8	2.04	0.09	46.24	7.1	50.41
0.7	3.0	2.10	0.49	9.00	3.7	13.69
0.9	3.9	3.51	0.81	15.21	4.8	23.04
0.4	6.9	2.76	0.16	47.61	7.3	53.29
1.9	14.5	27.55	3.61	210.25	16.4	268.96
1.1	22.5	24.75	1.21	506.25	23.6	556.96
0.9	21.7	19.53	0.81	470.89	22.6	510.76
(0.9)	19.5	17.55	0.81	380.25	20.4	416.16
0.7	15.7	10.99	0.49	246.49	16.4	268.96
1.2	16.0	19.20	1.44	256.00	17.2	295.84
(0.7)	17.1	11.97	0.49	294.41	17.8	316.84
0.4	5.9	2.36	0.16	34.81	6.3	39.69
0.	3.0	0.60	0.04	9.00	3.2	10.24
0.6	2.5	1.50	0.36	6.25	3.1	9.61
0.3	4.7	1.41	0.09	22.09	5.0	25.00
0.5	10.7	5.35	0.25	114.49	11.2	125.44
0.6	19.7	11.82	0.36	388.09	20.3	412.09

x	y	xy	x^2	y^2	(x+y)	$(x+y)^2$
0.4	22.7	9.08	0.16	515.29	23.1	533.61
1.0	16.7	16.70	1.00	278.89	17.7	313.29
0.3	14.3	4.29	0.09	204.49	14.6	213.16
0.7	13.1	9.17	0.49	171.61	13.8	190.44
0.3	1.0	0.30	0.09	1.00	1.3	1.69
3.9	14.1	54.99	15.21	198.81	18.0	324.00
Σx = 46.0	Σy = 555.6	Σxy = 643.19	Σx^2 = 79.98	Σy^2 = 9560.68	$\Sigma(x+y)$ = 601.0	$\Sigma(x+y)^2$ = 10925.68

n = 44 pairs of observations

Σx = 46.0

Σy = 555.6

Σxy = 643.19

Σx^2 = 79.98

Σy^2 = 9560.68

$\Sigma(x+y)$ = 601.0

$\Sigma(x+y)^2$ = 10925.68

The estimate of the Population regression equation is of the form $y_c = a + bx$, where y_c is the expected value of y for a given value of x. Here (a + bx) is called a linear estimator because it represents a linear relationship.

If the distributions of supopulations, y's are not specifical,

and if they possess equal variances corresponding to fixed x values, then according to the Gauss-Markov Theorem, the best linear estimator of a + bx can be obtained by the method of least squares. On the other hand, if the subpopulation distributions have been known or assumed to be normal then we may apply the maximum likelihood method to obtain an unbiased estimator of a + bx because we now know the form of the density functions of y's.

The procedure of estimating the population regression coefficients by the least squares method is to find a and b that will minimise $\Sigma(y - a - bx)^2$.

This requirement is met, according to the Gauss - Markov theorem, if a and b are determined by solving simultaneously the following set of normal equations :

1. $\Sigma y = na + b\Sigma x$
2. $\Sigma xy = a\Sigma x + b\Sigma x^2$

It turns out that the estimation of a + bx by the maximum likelihood method also requires the solution of a and b from the foregoing set of normal equations.

All quantities in the above normal equations can be computed from sample data. To solve for b, we multiply the first equation by Σx and the second by n, and then subtract the first result from the second so that a can be eliminated. As a result, we have

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Substituting the right side of the foregoing expression for b in either one of the two normal equations yields

$$a = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

Hence, $$b = \frac{(44 \times 643.19) - (46 \times 555.6)}{(44 \times 79.98) - (46 \times 46)} ;$$

$$\therefore \quad b = \frac{28300.36 - 25557.6}{3519.12 - 2116.00}$$

and $$a = \frac{(79.98 \times 555.6) - (46 \times 555.6)}{(44 \times 79.98) - (46 \times 46)} ;$$

$$\therefore \quad a = \frac{44436.888 - 25557.6}{3519.12 - 2116.00}$$

$$\therefore \quad a = \frac{2742.76}{1403.12} \qquad \therefore \quad a = \frac{18879.288}{1403.12}$$

$$\therefore \quad b = 1.9547 \qquad \therefore \quad a = 13.4552 .$$

The estimate for the population regression equation is therefore

$$y'_o = \text{expected value of } y = 13.4552 + 1.9547\,x$$

(y = sales; x = advertising)

∴ The predictive equation is $y_c = 13.4552 + 1.9547x$. But how good is it a predictive device? The answer to this question evidently rests upon the variability of the individual y values from the computed values of y associated with the x values. One way to visualise such dispersion is to draw a regression line through the points in the scatter diagram and to connect the former with the latter by vertical lines. To draw the regression line, it is only necessary to plot two y_c values with, perhaps, a third as a check. The regression line, of course, represents estimated values of y.

$$\text{At} \quad x = 0, \quad y = 13.4552 + (1.9547 \times 0) = 13.4552$$

$$\text{At} \quad x = 4, \quad y = 13.4552 + (1.9547 \times 4) = 21.2740 .$$

We see from the scatter diagram that the points deviate from the regression line. The numerical measure of such deviations is the estimate of the population variance, or the estimate of the population standard error, of the regression.

The sample standard deviation of regression, also called the standard error of estimate,

$$\sigma^2_{yx} = \frac{\Sigma y^2 - a\Sigma y - b\,\Sigma xy}{n-2}$$

This is an unbiased estimate of σ^2_{yx}. Here $(n - 2)$ degrees of freedom is used as the denominator. The two degrees of freedom lost correspond to the number of regression coefficients, two (a and b).

Hence, the sample standard deviation of regression

$$\sigma_{yx}^2 = \frac{9560.68 - (13.4552 \times 555.6) - (1.9547 \times 643.19)}{44 - 2}$$

$$\therefore \quad \sigma_{yx}^2 = \frac{9560.68 - 7475.70912 - 1257.243493}{42}$$

$$\therefore \quad \sigma_{yx}^2 = \frac{827.727387}{42} \qquad \therefore \quad \sigma_{yx}^2 = 19.7077795$$

Hence, the sample standard deviation of regression is

σ_{yx} = positive root of σ_{yx}^2 = positive $\sqrt{19.7077795}$ = 4.42

(The distribution of a is ignored, since a is not of much significance)

The estimated slope, b, of the predictive equation shows that there is a positive relationship between advertising x and sales y. But, b, being a sample statistic, is subject to sampling variability. (That is, because of chance variations, b may assume positive or negative values, but the population slope may actually be zero). If this is true, then the sample regression equation is of no value whatsoever as a predictive device. Thus, we must have knowledge of the nature of the sampling error in b and its size before we can use the sample regression equation with confidence.

The sampling distributionof b is normal if n ≥ 30 (n is actually equal to 44). If n < 30, b is subjected to students' t distribution. In our observations of advertising for, and sales of Coca-Cola, the sampling distribution of b is normal. The distribution

of b has as its variance

$$V(b) = \sigma_b^2 = \frac{\sigma_{yx}^2}{\Sigma(x - \bar{x})^2} .$$

The unbiased estimator of V(b) is

$$\sigma_b^2 = \frac{\sigma_{yx}^2}{\Sigma(x - \bar{x})^2} = \frac{\sigma_{yx}^2}{\Sigma x^2 - (\Sigma x)^2/n} = \frac{19.7077795}{79.98 - \frac{46 \times 46}{44}} = 0.618 .$$

The square root of the unbiased estimate of V(b) is the unbiased estimate of the standard error of b and is denoted as

$$\sigma_b = + \quad \sigma_b^2 = \sqrt{0.618} = 0.79$$

(computation of 95 / Confidence Interval - Interval Band for true regression line).

STEP 3 - The Computation of the Sample Correlation Coefficient.

When a sample of n pairs of observations is drawn, each x value is a random observation from the x population and each y value is a random observation from the y population, but the two are not necessarily independent; and when the assumption of bivariate normal population is met, the maximum likelihood estimator of the population correlation coefficient ρ, usually denoted as r, is obtained by the following expression

$$r = \rho = \frac{n\Sigma xy - \Sigma x\Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

which may vary from -1 through 0 to +1.

Then, the estimator for the population correlation coefficient,

$$r = \rho = \frac{(44 \times 643.19) - (46 \times 555.6)}{\sqrt{[(44 \times 79.98) - (46 \times 46)]\ [(44 \times 9560.68) - (555.6 \times 555.6)]}}$$

$$\therefore \quad r = \rho = \frac{28300.36 - 25557.60}{\sqrt{[3519.12 - 2116]\ [420669.92 - 308691.36]}}$$

$$\therefore \quad r = \rho = \frac{2742.76}{\sqrt{1403.12 \quad 111978.56}} = \frac{2742.76}{\sqrt{157119357.1072}} = \frac{2742.76}{12244.09}$$

$$\therefore \quad r = \rho = \frac{2742.76}{12244.09} = +0.224 .$$

$r = \rho = +0.224$ is an estimator for the population correlation coefficient ρ

$$\rho = \frac{\text{Covariance } (x,y)}{\sigma_x \sigma_y} = \frac{\text{Cov } (x,y)}{\sigma_x \sigma_y}$$

$$\text{i.e.} \quad \rho = \frac{E(x - \mu_x)\ (y - \mu_y)}{E(x - \mu_x)^2 \ \ E(y - \mu_y)} .$$

If $\rho = 0$, i.e. coo$(x,y) = 0$, there is no relationship between the variables. If ρ is between -1 and 0, the variables vary in opposite directions. If ρ is between 0 and +1, the variables are related such that they vary in the same direction.

For advertising on, and sales of, Coca-Cola and Fanta, $\rho = +0.224$, indicating that sales y depend on advertising x and sales y increases with increases in advertising x, and sales y decreases with decreases in advertising x.

The estimator r, like other statistics, is subject to sampling variations. When we obtain a positive or negative r, we may not be at all sure, because of sampling error, that the corresponding value of ρ is also positive or negative. This situation calls for a test of significance.

For the purpose of testing, we note that the sampling distribution of r varies with the value of ρ and the sample size. Under the assumption that the sample is drawn from a normal bivariate population, r tends to be distributed normally as the sample size increases. This tendency is much stronger for values of ρ that are close to 0 than for values of ρ that are close to -1 or +1 (When $\rho = 0$ and n is large, in other words, the distribution of r closely resembles a normal distribution.

The property that, when $\rho = 0$ and n is large, r is approximately normal, enables us to conduct the simplest and perhaps the most useful test on the null hypothesis that there is no correlation in the bivariate population, $\rho = 0$, against the alternative that $\rho \neq 0$. When $n < 30$, the statistic used to test this H_0 is $t = r \frac{n-2}{1-r^2}$ which is distributed as t_{n-2} if the null hypothesis is true.

In our sample, $n = 44$, i.e., $n > 30$, and the above statistic

explicitly the extent of the relationship between the two variables when the null hypothesis is rejected. If we desire to test a hypothesis that ρ has some value other than 0, or if we desire to construct a confidence interval for ρ, we may employ what is called the z transformation. That is, we may make a transformation of the asymmetric distribution of r into an approximately normal distribution as follows :-

$$Z_r = \frac{1}{2} \ln \frac{1 + r}{1 - r} = \frac{1}{2} \times 2.3026 \log_{10} \frac{1 + r}{1 - r} .$$

It can be shown that Z_r is approximately normally distributed with $E(Z_r) = Z_\rho$, and the estimated standard error is

$$\sigma_2 = \frac{1}{\sqrt{n - 3}} .$$

To test a hypothesis about ρ by r, we now have the testing statistic $Z = \frac{Z_r - Z_\rho}{\sigma_2}$ which is approximately $n(o,1)$.

To avoid the computations with logarithms, we employ a table of values for Z_r corresponding to various values of r as given by the statistical table for values of $Z = (\frac{1}{z}) \ln (\frac{1 + r}{1 - 4})$[138].

Now, suppose that we wish to test $H_0 : \rho = 0.25$ against $H_1 : \rho \neq 0.25$. We refer to the table and find

for $\rho = 0.25$, $Z_\rho = 0.25541$

for $r = 0.22$, $Z_r = 0.22366$

Hence,

$$Z = \frac{0.22366 - 0.25541}{\frac{1}{\sqrt{44-3}}} = \frac{-0.03175}{\frac{1}{\sqrt{41}}} = \frac{-.03175}{\frac{1}{6.4}} = \frac{-.03175}{0.15625}$$

$\therefore \quad Z = -0.203$

Since the critical value is greater than -1.96, H_0 is accepted at $\alpha = 0.05$.

We may safely assume, therefore, that the population correlation coefficient is positive and may be more than 0.25.

Confidence limits may be computed for Z_ρ as follows :

$$P\left(Z_r - Z_{\alpha/2}\sigma_2 < Z_\rho < Z_r + Z_{\alpha/2}\sigma_2\right) = 1 - \alpha$$

We shall now make a 95 percent confidence interval for the problem on hand : $(\sigma_z = 0.15625)$

$$0.22366 - 1.96(.15625) < Z_\rho < 0.22366 + 1.96(.15625).$$

$$\text{or} \quad -.08259 < Z_\rho < .52991$$

Corresponding to -.08259, $r = 0.00$

Corresponding to .52991, $r = 0.49$

Hence, the 95 percent confidence interval becomes

$$0 < \rho < 0.49$$

We may state, therefore, that the population correlation coefficient between advertising x and sales y lies between 0 and 0.49, and this statement is true 95 / of the time.

Step 4 - Population Coefficient of Determination (and sample)

We have defined the correlation coefficient of bivariate normal population as the degree of covariability between x and y. This definition led us to compute r as the maximum likelihood estimator of ρ. If we are concerned with a bivariate population whose distribution is unspecified, we can then define the population correlation coefficient as a measure of closeness-of-fit of the regression line. This new definition is based on the concept of Coefficient of determination. Consider the relationship between an individual value of y, y, and the population regression line of y on x, as well as the relationship between y and the mean of y, μ_y. The following relationship holds :

Total Error	=	Unexplained error +	Explained error
$y - \mu_y$		$(y - \mu_{yx})$	$(\mu_{yx} - \mu_y)$

$\mu_{yx} - \mu_y$ is called the explained error because its pattern is defined in the sense that it measures the amount of error which has been removed after the regression line has been fitted. The term $y - \mu_{yx}$ is called the unexplained error because it is a random quantity and is unpredictable : it measures that part of the total error which still remains or is unexplained after the regression line has been fitted.

The total sum of squares $\Sigma(y - \mu_y)^2$ can be partitioned into two independent parts : the unexplained sum of squares $\Sigma(y - \mu_{yx})^2$ and the explained sum of squares $(\mu_{yx} - \mu_y)^2$.

The coefficient of determination, ρ^2, is the ratio of the explained sum of squares to the total sum of squares. All values of ρ^2 range from 0 to 1. If $\rho^2 = 1$, each and every point in the scatter diagram will lie onthe regression line. If $\rho^2 = 0$, the points will be widely dispersed and will not resemble a straight line. ρ^2 is a measure that shows the relative reduction of the total error when a regression line is fitted.

The square of the population correlation coefficient is the population coefficient of determination. The square of the sample correlation coefficient is the sample coefficient of determination, r^2,

$$r = 0.224$$

$$r^2 = 0.050176 .$$

Hence, the points in the scatter diagram are quite widely dispersed, r^2 being low in value, since r^2 may be considered a measure of closeness-of-fit, there is low closeness-of-fit.

<u>Step 5</u> - The Analysis of Variance for Regression Analysis:

The linear regression model must be related to the linear model of the analysis of variance. To apply the analysis of variance terminology to regression, the quantity $\Sigma(y - y_c)^2$ may be thought of as the sum of squares which measures the unsystematic random disturbances and may be denoted as SSE. The quantity $\Sigma(y_c - \bar{y})^2$ is the sum of squares which measures the systematic (explained) variability due to regression and denoted as SSR. The sum of SSE and SSR is SST.

$$SST = \Sigma y^2 - n\left(\frac{\Sigma y}{n}\right)^2$$

$$SSR = b\left(\Sigma xy - \frac{1}{n}\Sigma x \Sigma y\right) \quad \text{and}$$

$$SSE = SST - SSR .$$

$$SST = 9560.68 - 44\left(\frac{555.6}{44}\right)^2 = 9560.68 - \frac{308691.36}{44}$$

$$\therefore \quad SST = 9560.68 - 7015.71 = 2544.97$$

$$SSR = 1.9547\left(643.19 - \frac{1}{44} \times 46 \times 555.6\right)$$

$$= 1.9547\left(643.19 - \frac{25557.6}{44}\right)$$

$$\therefore \quad SSR = 1.9547(643.19 - 580.90) = 1.9547 \times 62.29 = 121.758263$$

$$\therefore \quad SSE = 2544.97 - 121.758263 = 2423.211737 .$$

$(n = 44)$

ANALYSIS OF VARIANCE FOR COCA-COLA DATA

Source	SS	δ	MS
Regression	SSR = 121.758263	c = 1	MSR = SSR/1 = 121.758263
Error	SSE = 2423.211737	n - 2	MSE = SSE/n-2 = 57.695517
TOTAL	SST = 2544.97	n - 1	MST = SST/n-1 = 59.185

Obviously, the major part of the sum of squares is explained by residual error after fitting the regression line.

From the ANOVA table, we obtain the following statistics :

$$\sigma_{yx}^2 = MSE = 57.695517$$

$$\sigma_{yx} = \sqrt{57.695517} \approx 7.6$$

$$\sigma_b^2 = \frac{\sigma_{yx}^2}{\Sigma(x - \bar{x})^2} = 0.618 \quad \text{(Calculated previously)}$$

$$\sigma_b = + \sqrt{\sigma_b^2} \approx 0.79$$

$$r^2 = 1 - \frac{2423.211737}{2544.97} = 1 - 0.95219 = 0.04781 ,$$

which is quite close to the previously calculated value of $r^2 = 0.050176$.

When n is small, r^2 tends to be positively biased. To correct this bias, we compute the corrected coefficient of determination as follows :

$$\dot{r}^2 = 1 - \frac{MSE}{MST} = 1 - \frac{57.695517}{59.185} = 1 - 0.9748 = 0.0252$$

(However, in our sample, n = 44, and there is no real need to compute $\dot{r}^2$).

PART II - DISCUSSIONS REGARDING (TORCHES) DATA

The data regarding torches and cells (bulbs excluded) can be arranged in a manner which depicts quarterly advertising and corresponding quarterly sales.

MAY 1972 to APRIL 1973

Data on Torches and Cells produced by Geep Flashlight

Torches : x and y in Rs lakhs

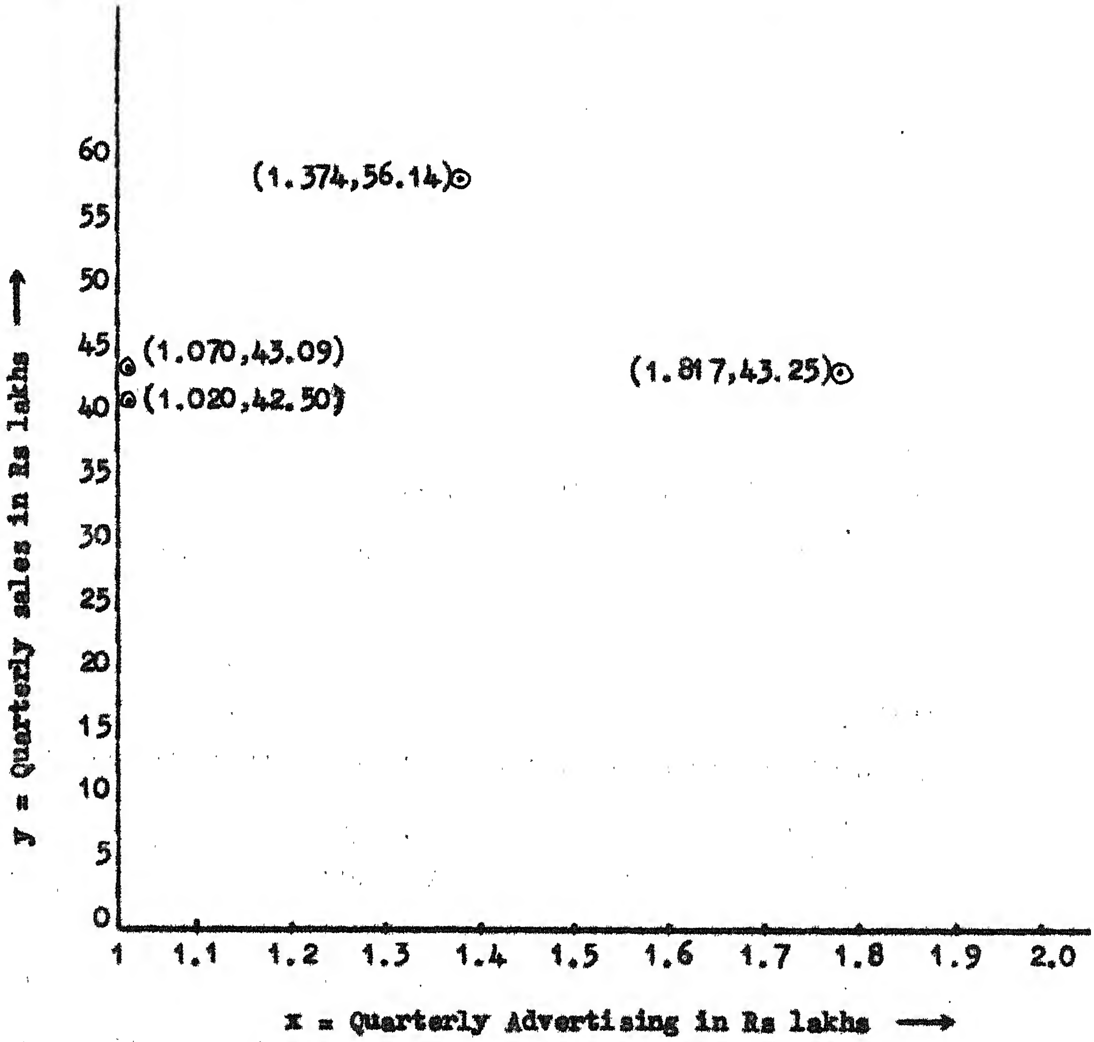
Time Period	Advertising (x)	Sales (y)
1^{st} Quarter	1.374	56.14
2^{nd} Quarter	1.817	43.25
3^{rd} Quarter	1.070	43.09
4^{th} Quarter	1.020	42.50

Cells : x and y in Rs lakhs

Time Period	Advertising (x)	Sales (y)
1^{st} Quarter	1.498	182.05
2^{nd} Quarter	1.104	191.52
3^{rd} Quarter	2.139	229.81
4^{th} Quarter	1.620	198.32

The scatter diagram for torches fails to depict any tendency for the points to fall along a line.

STEP 1 - Construction of scatter diagram for Torches



STEP - 2 : Linear Regression Analysis (for predictive Equation) for torches :

Sums required for Regression Computations.

x	y	xy	x^2	y^2	$(x+y)$	$(x+y)^2$
1.374	56.14	77.13	1.88	3151.69	57.514	3307.86
1.817	43.25	78.58	3.30	1870.56	45.067	2030.73
1.070	43.09	43.40	1.14	1856.74	44.160	1950.10
1.020	42.50	43.35	1.04	1806.25	43.520	1893.99
Σx	Σy	Σxy	Σx^2	Σy^2	$\Sigma(x+y)$	$\Sigma(x+y)^2$
= 5.281	= 184.98	= 242.46	= 7.36	= 8685.24	= 190.261	= 9182.68

n = 4 pairs of observations

$\Sigma x = 5.281$

$\Sigma y = 184.98$

$\Sigma xy = 242.46$

$\Sigma x^2 = 7.36$

$\Sigma y^2 = 8685.24$

$\Sigma(x+y) = 190.261$

$\Sigma(x+y)^2 = 9182.68$

The estimate of the Population regression equation is of the form $y_c = a + bx$, where y_c is the expected value of y for a given value of x

$$b = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{(4 \times 242.46) - (5.281 \times 184.98)}{(4 \times 7.36) - (5.281)^2}$$

$$\therefore \quad b = \frac{969.84 - 976.87938}{29.44 - 27.888961}$$

$$\therefore \quad b = -\frac{7.03938}{1.551039}$$

$$\therefore \quad b = -4.538$$

$$a = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma y}{n\,\Sigma x^2 - (\Sigma x)^2} = \frac{(7.36 \times 184.98) - (5.281 \times 184.98)}{(4 \times 7.36) - (5.281)^2}$$

$$\therefore \quad a = \frac{1361.4528 - 976.87938}{29.44 - 27.888961} = \frac{384.57342}{1.551039} = 247.9$$

$\therefore$ The Population regression equation for torches is

$$y_c = 247.9 - 4.538x \ .$$

We conclude that the data regarding torches indicates that sales actually decrease with advertising! Also, since at $x = 0$, $y_c = 247.9$ and at $x = 2$, $y_c = 247.9 - 9.076 = 238.824$, the Regression line is nearly parallel to the x-axis, and falls far above the points in the Scatter diagram. Such a Regression line has no practical meaning. There is no point in computing the sample standard deviation of regression, or the sampling distribution of b variance (However, the Regression line exists).

<u>STEP - 3</u> - The sample Correlation Coefficient for Torches :-

The maximum likelihood estimator of the population correlation coefficient ρ, denoted as r, is obtained from the expression

$$r = \rho = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$\therefore \quad r = \rho = \frac{(4 \times 242.46) - (5.281 \times 184.98)}{\sqrt{[(4 \times 7.36) - (5.281)^2][(4 \times 8685.24) - (184.98)^2]}} .$$

$$\therefore \quad r = \rho = \frac{969.84 - 976.87938}{\sqrt{[29.44 - 27.888961][34740.96 - 34217.6004]}}$$

$$\therefore \quad r = \rho = \frac{-7.03938}{\sqrt{1.551039 \quad 523.3596}} = \frac{-7.03938}{\sqrt{811.7511506244}} = \frac{-7.03938}{28.242}$$

$$\therefore \quad r = \rho = -0.24$$

Thus, the sales of torches and advertising on torches seem to be negatively correlated!

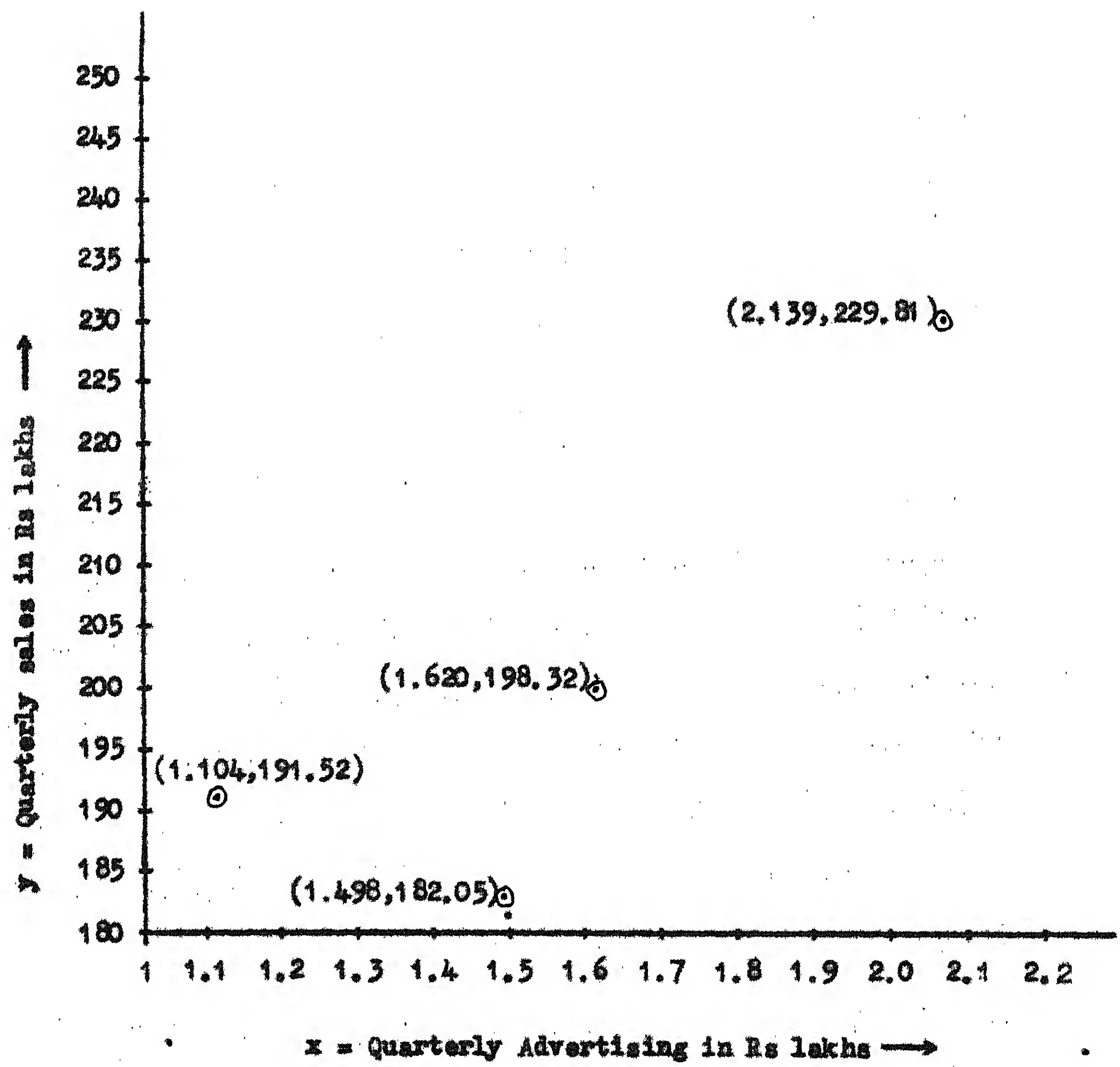
It is, however, plausible that correlation in the case of torches is negative because customers emphasize torch cells, not torches, since it is the cells which make the torches work. This is not true for a more complex equipment such as transistors, since cells do not by themselves determine the functioning of transistors - other factors are involved.

Thus, where the article is indispensable to the proper functioning of an equipment, sales and advertising are positively correlated. Otherwise, they may not be positively correlated.

The paucity of data on torches leads one to conclude that there is no effect of advertising on sales, for torches.

PART III - DISCUSSIONS REGARDING CELLS DATA

STEP - 1 - Construction of scatter diagram for Cells :



STEP - 2 - Linear Regression Analysis (for predictive Equation) for Cells.

Sums required for Regression Computations

x	y	xy	x^2	y^2	$(x + y)$	$(x + y)^2$
1.498	182.05	272.71090	2.244004	33142.2025	183.548	33672.25
1.104	191.52	211.43808	1.218816	36679.9104	192.624	37094.76
2.139	229.81	491.56359	4.575321	52812.6361	231.949	53777.61
1.620	198.32	321.27840	2.624400	39330.8224	199.940	39960.01
Σx	Σy	Σxy	Σx^2	Σy^2	$\Sigma(x+y)$	$\Sigma(x+y)^2$
$=6.361$	$=801.70$	$=1296.99097$	$=10.862541$	$=161965.5714$	$=808.061$	$=164504.63$

Hence n = 4 pairs of observations

$\Sigma x = 6.361$;

$\Sigma xy = 1297.99097$;

$\Sigma y^2 = 161965.5714$;

$\Sigma y = 801.7$;

$\Sigma x^2 = 10.862541$;

$\Sigma(x + y) = 808061$;

$\Sigma(x + y)^2 = 164504.63$

The estimate of the population regression equation is of the form $y_c = a + bx$, where y_c is the expected value of y for a given value of x.

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$\therefore \quad b = \frac{(4 \times 1296.99097) - (6.361 \times 801.7)}{(4 \times 10.862541) - (6.361)^2}$$

$$\therefore \quad b = \frac{5187.96388 - 5099.6137}{43.450164 - 40.462321}$$

$$\therefore \quad b = \frac{88.35018}{2.987843} = 29.5698$$

$$a = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \; .$$

$$\therefore \quad a = \frac{(10.862541 \times 801.7) - (6.361 \times 801.7)}{(4 \times 10.862541) - (6.361)^2}$$

$$\therefore \quad a = \frac{8708.4991197 - 5099.6137}{43.450164 - 40.462321}$$

$$\therefore \quad a = \frac{3608.8854197}{2.987843} = 1207.8565 \; .$$

Hence the estimate for the population regression equation is

$$y_c = 1207.8565 + 29.5698\,x \quad \text{(for cells)}$$

The Regression line can be obtained by plotting for two values of x, say $x = 0$ and $x = 2$ and joining these two points. On the plotted scattered diagram, diagrammatic representation of the Regression line is difficult, since it falls far above the x-axis.

STEP - 3 - The computation of the sample correlation coefficient for cells.

The maximum likelihood estimator of the population correlation coefficient ρ, denoted as r, is obtained from the expression

$$r = \rho = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$\therefore \quad r = \rho = \frac{(4 \times 1296.99097) - (6.36 \times 801.7)}{\sqrt{[(4 \times 10.862541) - (6.361)^2][(4 \times 161965.5714) - (801.7)^2]}}$$

$$\therefore \quad r = \rho = \frac{5187.96388 - 5099.6137}{\sqrt{(43.450164 - 40.462321)(647862.2856 - 642722.89)}}$$

$$\therefore \quad r = \rho = \frac{88.35018}{\sqrt{2.987843 \quad 5139.3956}} = \frac{88.35018}{\sqrt{15355.7071676908}}$$

$$\therefore \quad r = \rho = \frac{88.35018}{122.08510} = +0.72367$$

It is obvious that, for cells, sales and advertising are positively correlated, with a correlation coefficient of +0.72367.

Since n is only 4, we do not calculate the confidence interval for ρ, since $\sigma_2 = \frac{1}{\sqrt{n - 3}} = \frac{1}{\sqrt{4 - 3}} = 1$, and the results may not be significant in a real sense.

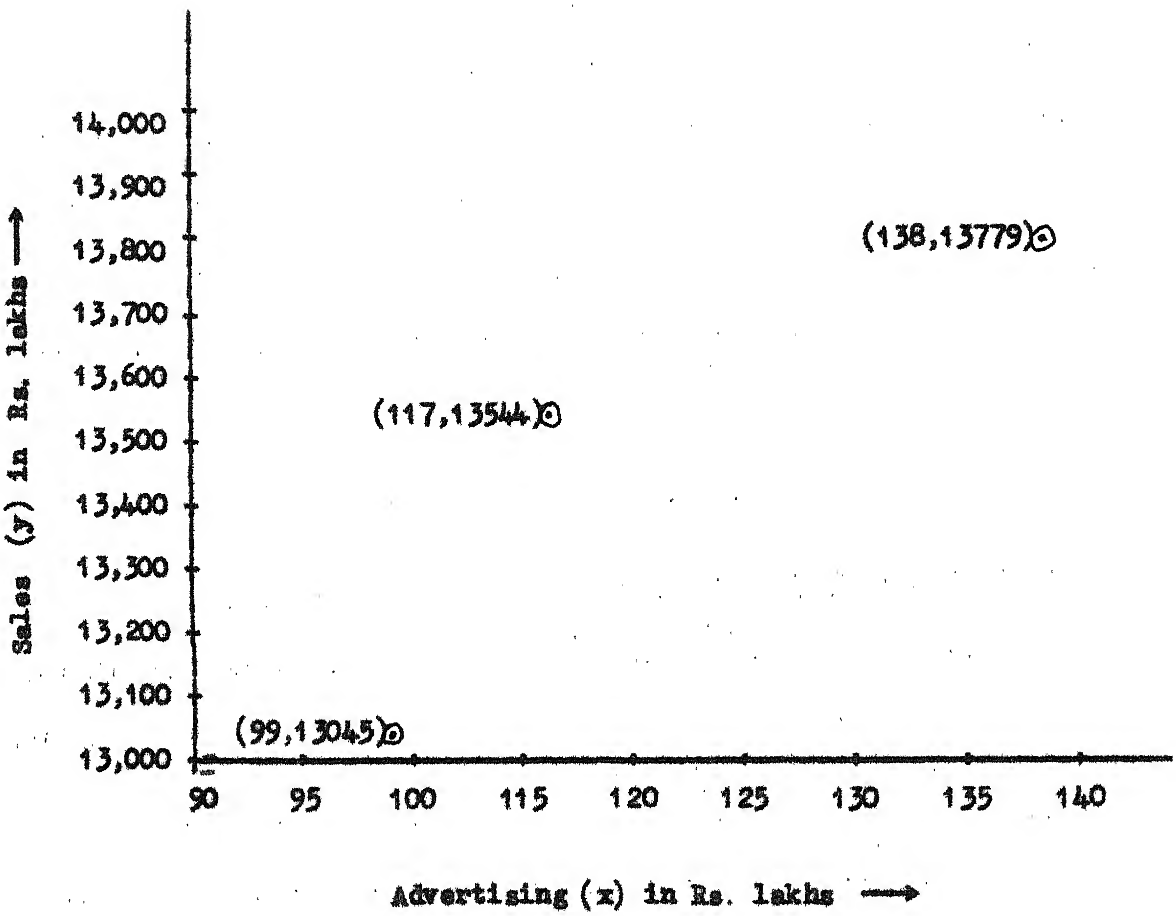
STEP - 4 - The sample coefficient of determination (for data on cells) is $r^2 = (.72367)^2 = 0.5236982689$. Hence, the points in the scatter diagram are not widely dispersed about the regression line. There is fairly good closeness-of-fit.

There is no need for an independent ANOVA analysis, (since n is only 4), as was done in the case of Coca-Cola data.

PART IV - DISCUSSIONS ON D.C.M. DATA

STEP 1 - Construction of scatter diagram for D.C.M. Products. To the nearest lakh, the figures for Advertising and Sales are

Year	Advertising (x) (In Rs.lakhs)	Sales (y) (In Rs.lakhs)
1971 - 72	117	13544
1972 - 73	99	13045
1973 - 74	138	13779



STEP 2 - Linear Regression Analysis (for predictive equation) for D.C.M. products.

Sums required for Regression Computations

x	y	xy	x^2	y^2	$(x + y)$	$(x + y)^2$
117	13544	1584648	13689	183439936	13661	186622921
99	13045	1291455	9801	170172025	13144	172764736
138	13779	1901502	19044	189860841	13917	193682889

Σx	Σy	Σxy	Σx^2	Σy^2	$\Sigma(x+y)$	$\Sigma(x+y)^2$
=354	=40368	=4777605	=42534	=543472802	=40722	=553070546

Hence, n = 3 pairs of observations

$\Sigma x = 354$

$\Sigma y = 40368$

$\Sigma xy = 4777605$

$\Sigma x^2 = 42534$

$\Sigma y^2 = 543472802$

$\Sigma(x + y) = 40722$

$\Sigma(x+y)^2 = 553070546$.

The estimate of the population regression equation is of the form $y_c = a + bx$, where y_c is the expected value of y for a given value of x.

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$\therefore \quad b = \frac{(3 \times 4777605) - (354 \times 40368)}{(3 \times 42534) - (354)^2}$$

$$\therefore \quad b = \frac{14332815 - 14290272}{127629 - 125316}$$

$$\therefore \quad b = \frac{42543}{2313} = 18.393$$

$$a = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

$$\therefore \quad a = \frac{(42534 \times 40368) - (354 \times 40368)}{(3 \times 42534) - (354)^2}$$

$$\therefore \quad a = \frac{1857885 12 - 14290272}{127629 - 125316}$$

$$\therefore \quad a = \frac{171498240}{2313} = 74145.37$$

Thus, the estimate for the population regression equation for D.C.M. products is $y_c = 74145.37 + 18.393\,x$. Thus, for D.C.M. sales definitely increase with an increase in advertising A definite regression line is obtained.

The standard error of estimate, or the sample standard derivation of regress,

$$\sigma_{yx}^2 = \frac{\Sigma y^2 - a\Sigma y - b\Sigma xy}{n-2}$$

$$\therefore \quad \sqrt{\sigma_{yx}^2} = \frac{543472802 - (74145.37 \times 40368) - (18.393 \times 4777605)}{3-2}$$

But $\sqrt{\sigma_{yx}^2}$ need not be computed, since the small number of pairs of observations $(n = 3)$ obviates the practical significance of such a computation.

Since $n < 30$, the sampling distribution of b is subjected to students' t distribution. The distribution of a is ignored.

STEP 3 - The computation of the sample correlation coefficient for D.C.M. data.

The maximum likelihood estimator of the population correlation coefficient ρ, denoted as r, is obtained from the expression

$$r = \rho = \frac{n\Sigma xy - \Sigma x\Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$\therefore \quad r = \rho = \frac{(3 \times 4777605) - (354 \times 40368)}{\sqrt{[(3 \times 42534) - (354)^2][(3 \times 543472802) - (40368)^2]}}$$

$$\therefore \quad r = \quad = \frac{14332815 - 14290272}{\sqrt{(127629 - 125316)(1630418406 - 1629575424)}}$$

$$\therefore \quad r = \rho = \frac{42543}{\sqrt{(2313) \quad (842982)}}$$

$$\therefore \quad r = \rho = \frac{42543}{\sqrt{1949817366}}$$

$$\therefore \quad r = \rho = \frac{42543.0}{42743.3} = 0.9953$$

It is obvious that there is a very high positive correlation

between advertising for, and sales of, D.C.M. products, the correlation being as high as + 0.9953.

STEP 4 - Sample Coefficient of determination for D.C.M. data.

The sample coefficient of determination $r^2 = (.9953)^2$

$\therefore \ r^2 = 0.99062209$. There is good closeness-of-fit. If more points in the scatter diagram had been available, such points would have been strongly concentrated around the regression line. ANOVA analysis for D.C.M. data has not been done, since there is no appreciable information to be gained from σ_{yx}^2 and σ_b^2. Also, the corrected coefficient of determination r^2 will not be significantly different from the sample coefficient of determination r^2.

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PART VI

C O N C L U S I O N

CONCLUSIONS

1. For the soft-drink "Coca-Cola", the relationship between advertising expenditure x and sales Revenue y is $y = 13.4552 + 1.9547x$.

2. For the soft-drink "Coca-Cola", advertising expenditure and sales Revenue are positively correlated, the correlation coefficient being + 0.224.

3. For the soft drink "Coca-Cola", the correlation coefficient between advertising expenditures and sales revenues lies between 0 and + 0.49 for 95 / of the time.

4. For the soft-drink "Coca-Cola", the coefficient of determination is + 0.050176 and there is low closeness-of-fit of the regression line.

5. For cell-less electrical torches, advertising expenditure and sales revenue are negatively correlated, the correlated coefficient being - 0.24.

6. For cell-less electrical torches, the relationship between advertising expenditure x and sales revenue y is $y = 247.9 - 4.538x$.

7. For electrical cells, the relationship between advertising expenditure and sales revenue y is $y = 1207.8565 + 29.5698x$.

8. For electrical cells, advertising expenditure and sales revenue

are positively correlated, the correlation coefficient being + 0.72367.

9. For electrical cells, the coefficient of determination is + 0.5236982689 and there is fairly good closeness-of-fit of the regression line.

10. For D.C.M. products, the relationship between advertising expenditure x and sales revenue y is $y = 74145.37 + 18.393x$.

11. For D.C.M. products, advertising expenditures and sales revenues are positively correlated, the correlation coefficient being + 0.9953.

12. For D.C.M. products, the coefficient of determination is + 0.99062209, and there is good closeness-of-fit of the regression line.

13. For each product, there is a certain sales level which would exist in the absence of advertising, but increases in sales beyond the said level are a consequence of advertising.

14. Advertising has significant, quantifiable and predictable influences on sales.

PART VII

S U M M A R Y

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A critical analysis of studies conducted in the field of Advertising - sales Interaction was performed. In particular, investigations conducted by Magee (U.S.A., 1953), Kuehn (U.S.A., 1955), Zentler and Ryde (U.S.A., 1956), Vidale and Wolfe (U.S.A., 1957), Starch (U.S.A., 1958), Telser (U.S.A., 1962), Twedt (U.S.A., 1963), Lucas and Britt (U.S.A., 1963), Palda (U.S.A., 1964), Quandt (U.S.A., 1964), Amstutz (U.S.A., 1967), Couffignal (FRANCE, 1967), Marantz (FRANCE, 1967), Doyle (U.S.A., 1968), Montgomery and Urban (U.S.A., 1969), Schultz (U.S.A., 1970), Schmalensee (U.S.A., 1972), Sethi (CANADA, 1973), Nakanashi (U.S.A., 1973), and by Rao and Miller (U.S.A., 1975) were scrutinised, evaluated and discussed. The author subjected data regarding advertising outlays on, and sales of, soft-drinks, torches and cells, and D.C.M. products to statistical analysis, and concluded that Advertising has significant, quantifiable and predictable influences on sales.

PART VIII

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